Model Free Jump Measures and Interest Rates: Common Patterns in US and UK Monetary Policy around Major Economic Events

Januj Juneja and Kuntara Pukthuanthong

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Abstract

We employ model-free jump measures to study monetary policy operations in the US and UK around major economic events by exploiting the relationship between jumps, interest rates, and macroeconomic news releases related to monetary policy. In our analysis, we explicitly account for the timing of jumps in US and UK interest rates and the correlation across jumps in the same two interest rates and whether these match FOMC/MPC news releases. We find that FOMC news releases lag jumps in US interest rates, but lead jumps in UK Gilt rates. Overall, our analysis suggests that US T-bills react to information in the aforementioned news releases before their announcement while UK Gilt yields react after them and that the Fed and Bank of England react similarly around major economic events.

JEL Classification: E52; C140; E430

Key Words: Non-parametric methods; Monetary Policy; Interest Rates
1. **INTRODUCTION**

It is widely known that jumps exist in interest rate dynamics (Das 2002; Johannes 2004; Lee and Mykland 2008; Dungey, McKenzie, and Smith 2009; Dungey and Hvozdyk 2012). Pukthuanthong and Roll (2015) note that in general there could be several different causes of jumps including political upheaval, extreme events, and shocks to commodity prices. There is a plethora of evidence suggesting that macroeconomic news announcements cause jumps in interest rates (Das 2002; Hamilton and Jordà 2002; Farnsworth and Bass 2003; Andersen, Bollerslev, and Diebold 2007; Heidari and Wu 2010; Jiang, Lo, and Verdelhan 2011; Lahaye, Laurent, and Neely 2011; Lee 2012; Savor and Wilson 2013). Thus, scholars have been developing analytical frameworks for incorporating jumps into interest rate models for decades (Ahn and Thompson 1988; Anderson and Lund 1997; Ahn, Dittmar, and Gallant 2002; Das 2002; Finnerty 2005; Piazzesi 2005; Wu and Zeng 2006; Jiang and Yan 2009; Mancini and Renò 2011). Recently, several researchers have proposed model-free measures (Barndorff-Nielsen and Shephard 2006; Jiang and Oomen 2008; Lee and Hannig 2010) for the study of jumps in interest rates claiming that such non-parametric jump measures yield estimation advantages relative to traditional approaches which rely on sophisticated analytical tools for implementation (Johannes 2004; Lee and Mykland 2008). Given these advantages, researchers have begun using model-free measures to study the relationship between jumps and macroeconomic news announcements (Lee 2012; Léon and Sebestyén 2012; Gilder et al. 2013).
The objective of this paper is to use model-free jump measures to study monetary policy operations conducted in the US and UK over the period 1997-2013. We select the period 1997-2013, since as a period, it saw several major economic events and shocks that impacted the global economy (see, e.g., Kaeck and Alexander (2013) for a list of important economic events that occurred during this period). In particular, we intend to examine whether the Fed and the Bank of England conduct monetary policy operations in a similar manner around major economic events. We limit our study to only the US and UK because the two countries have a long history of mutual cooperation and partnership (Mix 2013). Miles (2013) provides illustrative evidence that although the Fed and Bank of England have different formal objectives, they both set monetary policy in similar ways because they place a strong emphasis on real variables (e.g., output and employment).³

We examine whether the timing of jumps in the US and UK interest rates and correlation in jumps in the same two interest rates match macroeconomic news releases related to monetary policy. Although many news announcements have been shown to be significant in the literature, since our focus is on monetary policy, we only focus on FOMC (Federal Open Market Committee) meeting dates, which include one-day & two-day meetings, conference calls, and unscheduled meetings, and MPC ( Monetary Policy Committee) meeting dates, which include two-day meetings and unscheduled meetings.

We detect jumps using model-free jump measures applied to daily data from the US and UK Treasury markets. We employ jump measures proposed by Jiang and Oomen (2008, henceforth JO) and Lee and Mykland (2008, henceforth LM). We recognize that there have been
several methods proposed in the literature (Aït-Sahalia and Jacod 2009; Andersen, Bollerslev, and Debrev 2007; Gobbi and Mancini 2007; Bollerslev, Law, and Tauchen 2008; Christensen, Oomen, and Podolskij 2010; Jacod and Todorov 2009; Anderson, Dobrev, and Schaumburg 2012; Aït-Sahalia, Cacho-Diaz, and Raeven 2013), but we limit our selection to these two jump measures for the following reasons. Most generally, we restrict our selection to those measures that are based upon the bi-power variation test, which has been widely implemented in the literature (Jiang and Oomen 2008). We include the JO measure because not only is it easy to implement and similar to the bi-power variation measure, but it also demonstrates more power than the bi-power variation test under many circumstances (Jiang and Oomen 2008). We feel that this latter characteristic of the JO measure potentially yields a nice contrast to the bi-power variation test. Employing the JO measure and a measure based upon the bi-power variation test such as LM or Barndorff-Nielsen and Shephard (2006) enables us to broaden the implications of our findings. For the measure based upon the bi-power variation test, we only study the LM measure because JO compare their findings to Barndorff-Nielsen and Shephard (2006) throughout their study and so studying it would not add value. Although the LM measure is the only measure, which allows us to estimate jumps on a daily basis, Anderson, Bollerslev, Frederiksen, and Nielsen (2010) propose a sequential procedure, which can be employed to identify jumps in interest rates collected at the daily frequency. We utilize this sequential method to estimate the JO measure using daily data.4

In their implementation of statistical jump measures, many scholars seem to focus exclusively on very high frequency data (Bates 2000; Pan 2002; Johannes 2004; Dungey,
McKensie, and Smith 2009; Tauchen and Zhou 2011). Asset prices are usually assumed to evolve in continuous time, and jumps are envisioned as instantaneous discontinuities. For a jump to occur between two interest rate series, it must happen at precisely the same instant. However, it is less clear that non-mathematically inclined investors care much about whether jumps occur in two interest rates at precisely the same instant. A few professional investment organizations monitor markets more or less continuously, but the vast majority are less attentive; as long as jumps occur within the investment review period, practitioners can successfully implement their trading strategies. Consequently, following the approach of Pukthuanthong and Roll (2015), we test for correlations in jumps using formulations that are originally developed for continuous time methods using measurable calendar periods. We take this measurable calendar period to be days, since investors implementing trading strategies require some time to devise their strategies. Hence, our jump measures are computed based upon data collected at a daily frequency.

Additionally, very recently, researchers have begun to use daily data to study jumps (Johannes 2004; Hausman and Wongswan 2011; Leôn and Sebestyên 2012; Lucca and Moench 2015; Pukthuanthong and Roll 2015). Wongswan (2009) and Hausman and Wongswan (2011) conclude that estimates from daily data are very close to those based upon high-frequency data. Leôn and Sebestyên (2012) use daily data to implement new measures to characterize European Central Bank monetary policy and find that their measures are superior to traditional measures for computing surprises in monetary policy. As they use daily data to construct “superior” measures, we feel that their findings complement and even strengthen the approach taken in our paper.
We find that jumps are not significantly correlated but jump correlations are the strongest when FOMC unscheduled meetings and conference calls are held. Given that FOMC unscheduled meetings and conference calls are held in the midst of important economic events, the implication is that while jumps across US and UK interest rates are idiosyncratic, they are significantly correlated in the presence of important economic events. Our findings demonstrate that on the whole, FOMC meetings, especially unscheduled ones and conference calls, lagged US T-bill returns but led UK Gilt returns. Employing the LM measure, we find that FOMC unscheduled meetings and conference calls lagged jumps in US T-bill returns by 4 days and led UK Gilt returns by 3.7 days while according to the JO measure, FOMC unscheduled meetings and conference calls lagged jumps in US T-bill returns by 4.3 days and led UK Gilt returns by 3 days. Our findings suggest that the US T-bill market is efficient and US T-bill returns jump before the announcement of the meetings while UK Gilt returns jump after the announcement of the meeting. We also find that correlations in jumps of the returns of UK and US interest rates were the strongest around important global economic events. Some examples of such events include the Brazilian Currency Crisis and Russian Financial Crisis, both occurring in Autumn 1998. We also find that the Global Financial Crisis of 2007-2010 occurred when correlations in jumps in UK and US interest rates were the strongest over our sample period. From our results, we interpret that around major economic events, the Fed and the Bank of England broadly focus on the implementation of similar policies relating to the restoration of stability (see, e.g., Miles 2013; Viñal, Blanchard, and Bayoumi 2013) in their financial markets.
The remainder of the paper is organized as follows. We describe the dataset in Section 2. In Section 3, we provide an overview of the jump measures studied in this paper. In Section 4, we study whether the timing of jumps in UK and US interest rates and the correlation in jumps in the same two series match the FOMC/MPC meetings announcement releases. In Section 5, we examine whether or not the periods, which contribute the most strongly to the correlation in jumps in US, and UK interest rates contain important economic events. Finally, Section 6 concludes.

2. DATA

2.1 Summary Statistics for Market Rates

The dataset we use in this study comprises daily observations of secondary market rate US Treasury Bills and UK Gilts yields with a three-month maturity from January 2, 1997 through December 31, 2013. The three-month US Treasury Bill data is from Release Fed H.15, while the three month UK Gilt yield data is from Bloomberg. The descriptive statistics for each variable are shown in Table 1.

***Table 1***

The minimum of 0.00% for the three month T-bill occurred during the financial crisis: December 10, 2008, December 18, 2008, December 24, 2008, September 22, 2011, October 5, 2011, December 15 and 16 2011, and September 26, 2013. Excluding those days, the minimum of all other days in the sample is 0.01%.

We decide to use U.S. Treasury Bill rates and UK Gilt rates as our selection for the short-term interest rate rather than the Federal Funds rate for the US and the Bank rate for the UK,
respectively. Microstructure noise effects associated with the settlement of the Fed Funds rate have been known to spuriously create jumps in the daily Fed Funds Rate (Hamilton 1996; Johannes 2004). Additionally, Treasury bill data has small idiosyncratic effects, is very liquid, and has small bid-ask spreads (Johannes 2004). For our sample period, the correlation between the three-month US T-bill and the Fed Funds rate is about 0.992. Similarly, we decided to use the three-month UK Gilt rate instead of the Bank rate. For our sample period, the correlation between the Bank rate and three month UK Gilt is about 0.997 over the sample period under examination.

Additionally, our sample consists of 75 one-day meetings, 62 two-day meetings, 11 conference calls, and 14 unscheduled meetings for the Fed’s FOMC. Over that same period, the Bank of England’s MPC had 194 two-day meetings. During our sample period, the UK had three unscheduled meetings: September 18, 2001, January 21, 2008, and October 8, 2008. In total, the MPC met 197 times.

Data on announcements corresponding to FOMC meetings comes from the U.S. Federal Reserve bank website. If one were to browse the Fed website, one would notice that the meeting calendar is posted well before scheduled meetings (i.e., one-day meetings and two-day meetings) are held. Hence, as market events, one-day and two-day meetings should marginally affect the probability of a change in rates (Heidari and Wu 2010). On occasion, the Fed holds unscheduled meetings and/or conference calls to be able to make more timely policy decisions. These meetings last one day and are posted after the fact on the Fed website. Since information relevant to these meetings is not posted until after they occur, one could imagine that they should signal more information than a scheduled meeting.
Similar to scheduled meetings held by the US FOMC, one would not expect an information surprise from two-day meetings because the MPC’s 2-day meeting normally is held during the first ten days of the month.\textsuperscript{9} The MPC does not have one-day meetings. Although infrequent, the Bank of England holds unscheduled meetings as well. Data on unscheduled meetings are obtained from the Inflation Reports which are also posted on the Bank of England website.\textsuperscript{10}

2.2 Summary Statistics for Correlations of Continuously Compounded Returns

Although our focus is on detecting jumps in interest rates, we begin by reporting correlations obtained from logarithmic or continuously compounded returns; these will prove to be useful as a basis for comparison. We choose to calculate continuously compounded returns because we implement the JO and LM measures to detect jumps and their measures are derived based upon continuously compounded returns. To calculate continuously compounded returns for both T-bills and Gilts, which are zero-coupon yields, we first need to convert bond yields into bond prices and then, using these bond prices we calculate the logarithmic return on day $t$ as the natural logarithm of the ratio of the bond price on day $t$ to the bond price on day $t-1$. If we follow existing literature and denote the bond price on day $t$ as $X(t)$, then the aforementioned quantity can be written more precisely as $\log \left( \frac{X(t)}{X(t-1)} \right) = \log(X(t)) - \log(X(t-1)) = d\log(X(t))$. This quantity is known as the continuously compounded return (or logarithmic return).\textsuperscript{11} Then, for each instrument, we compute returns on a semimonthly and monthly basis by adding together the (log) daily increments. Simple product moment correlations of 3-month T-bill and Gilt continuously compounded returns, given in Table 1, are 0.32 and 0.17 with a p-value of 0 and
0.001 on a monthly and semimonthly basis, respectively. The correlation between the two series is significant at the 1% level. To gain more insight into the nature of the correlation between the two series, we determine the calendar periods which contribute most strongly to the correlation between interest rates in the US and the UK which are taken to be the period with the largest correlation (computed as the demeaned product of continuously compounded returns).

Table 2 provides a list of the most influential observations for the correlation between continuously compounded returns corresponding to 3-month T-bill and Gilt rates. The top two influential periods are October and January 2008 on a monthly basis, and the 2nd half of December 1998 and the 1st half of October 2008 on a semimonthly basis. These periods except for the 2nd half of December 1998 occurred during the Global Financial Crisis and are defined by NBER (National Bureau of Economic Research) as contraction periods; such periods are characterized by elevated levels of risk (see, e.g., Pais and Stork (2013)). Moreover, the unscheduled meeting that the Bank of England conducted on October 8, 2008 represents an influential period. The Bank of England’s unscheduled meetings all followed either unscheduled meetings or conference calls held by the US Federal Reserve Bank. The unscheduled meeting held by the Bank of England on September 18, 2001 followed conference calls held by the US Federal Reserve Bank that took place on September 13, 2001 and September 17, 2001. The unscheduled meeting held by the Bank of England on October 8, 2008 followed the US Federal Reserve Bank unscheduled meeting that occurred on October 7, 2008.

*** Table 2 ***
The most influential periods for the correlations between continuously compounded returns to T-bills and Gilts occur during NBER recession periods implying that continuously compounded returns to T-bills and Gilt rates are most strongly correlated during recessionary periods. Indeed, as we demonstrate in subsequent sections, jumps in T-bills and Gilts are strongly correlated around important economic events that occurred during these periods.

3. MODEL-FREE JUMP MEASURES AND INTEREST RATES

3.1 Overview of Jump Measures Applied in this Study

We begin our analysis by using the LM and JO measures to detect jumps in the dataset described in Section 2. Both measures imply a strong presence of jumps in both interest rate series. As these findings are consistent with the previous literature and hence are well known, we do not show them. They are available upon further request. Below, we provide an overview of each measure. In Section 4, we employ these measures to examine the extent to which jumps in UK and US interest rates are correlated around major economic events.

3.2 The Lee and Mykland (2008) (LM) Statistic

The derivation of the LM test statistic starts from a one-dimensional asset return process with fixed complete probability space \((\Omega, \mathcal{F}, \mathbb{P})\), where \(\{\mathcal{F}_t; t \in [0, T]\}\) is a right-continuous information filtration and \(\mathbb{P}\) is a measure used to generate the data.\(^\text{13}\) This asset return process is specified as follows in the presence of jumps

\[
d log X(t) = \alpha(t) dt + \beta(t) dW(t) + \gamma(t) dJ(t)
\]

Eq. (1)
In Equation (1) above, $d\log X(t)$ is the continuously compounded return and $X(t)$ is the asset price, $\alpha(t)$ and $\beta(t)$ are $\mathcal{F}_t$-adapted processes, $\gamma(t)$ is a predictable process, $df(t)$ is a counting process, and $dW(t)$ is a standard Brownian motion. For simplicity, we adopt the same notation as LM for the counting process. Based upon this stochastic process for the asset return dynamics, LM define their test statistic as the ratio of realized return to estimated volatility, and use the bipower variation for the estimated instantaneous volatility.

The LM test statistic, which tests whether or not there is a jump over the period $t_{i-1}$ to $t_i$, is approximately

$$L = \log X(t_i)/X(t_{i-1}) / \sqrt{\frac{1}{K-2} \sum_{j=i-K}^{i-2} \left| \log X(t_j)/X(t_{i-1}) \right| |X(t_{i-1})/X(t_{i-2})|}$$

In the approximate expression for the LM test statistic above, the realized return over the period $t_{i-1}$ to $t_i$ is given by $\log X(t_i)/X(t_{i-1})$ and the bipower variation, which is computed as the sum of the products of consecutive absolute value of returns over some sub-period $K$, is given by $\sqrt{\frac{1}{K-2} \sum_{j=i-K}^{i-2} \left| \log X(t_j)/X(t_{i-1}) \right| |X(t_{i-1})/X(t_{i-2})|}$. Taking the ratio of these two terms leads to the test statistic value. In Theorem 1 (Page 2541) of their paper, LM demonstrate that, asymptotically, their test statistic approximately follows a normal distribution, and specify the conditions under which this relationship becomes exact (see LM for more information).

To compute the LM statistic, we classify each period into those that are likely to contain jumps and those that are not. After classifying each sample period as jump or non-jump, we then construct a 2x2 contingency table for a pair of 3-month T-bill and Gilt rates, depicted below for
interest rate i and j, \( N_{ij} \) is the number of months in column i and row j and their sum is N, the total number of months with concurrent observations for interest rates i and j.

<table>
<thead>
<tr>
<th>Jump in i</th>
<th>Jump in j</th>
<th>No jump in j</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_{1,1}</td>
<td>N_{2,1}</td>
<td></td>
</tr>
<tr>
<td>N_{1,2}</td>
<td></td>
<td>N_{2,2}</td>
</tr>
</tbody>
</table>

If there is no connection between the jumps that occur in interest rates i and j, then the “expected” number of months in the top left cell is \( E_{1,1} = (N_{1,1}+N_{1,2})(N_{1,1}+N_{2,1})/N \), the product of the marginals, and so on, for each of the other cells. The Chi-square statistic is

\[
\chi^2 = \sum_{i,j} \frac{(N_{i,j} - E_{i,j})^2}{E_{i,j}},
\]

Eq. (2)

which has one degree of freedom. We implement equation (2) using the data described in Section 2 and find a mean Chi-square value of 0.10 on a monthly basis and 1.59 on a semimonthly basis. See Table 1. Both Chi-square values are less than the expected value of 2.0 under the null hypothesis (number of jumps in interest rate i is independent of number of jumps in interest rate j; thus, under the LM measure, jumps in continuously compounded returns corresponding to both Gilts and T-bill are independent.

### 3.3. The Jiang and Oomen (2008) (JO) Statistic

The derivation of JO starts from the following assumption about the logarithm of the asset price in the presence of jumps.

\[
dy_t = \mu_t dt + \Sigma_t^{1/2} dB_t + \lambda_t dJ_t
\]

Eq. (3)
In equation (3) above, \( y_t = \ln S_t \), where \( S_t \) is the level of the price process, \( \mu_t \) is the instantaneous drift, \( \Sigma_t \) is the instantaneous variance when there is no jump, \( \lambda_t \) is a random variable that represents jumps in the price of the asset, \( B_t \) is a standard Brownian motion and \( J_t \) is a counting process. Additionally, no functional specifications are imposed on coefficients \( \mu_t \), \( \Sigma_t \), or \( \lambda_t \). Applying Itô’s Lemma to Equation (3) yields the price process in levels, which is necessary for the construction of the test statistic. The difference between the simple or arithmetic return (computed in levels) and the log returns forms the basis for the construction of the JO test because it relies on the fact that this difference captures one-half of the integrated variance in the continuous-time limit (see, e.g., Neuberger 1994 or JO). This relation also forms the basis for the variance swap replication strategy (Neuberger 1994), which yields much of the underlying properties and logic for the test. The profit/loss to such a strategy accrues to a quantity that is proportional to the realized variance, and in this case replication of the position is exact. However, in the presence of jumps, the strategy fails and the replication error is completely determined by realized jumps. In line with this reasoning, the JO test compares the realized variance to the difference between simple and log returns. When jumps are absent, this difference is zero, but in the presence of jumps it represents the replication error of the variance swap strategy, which provides the test with the power to detect jumps. Under the null hypothesis of no jumps and in Theorem 2.1 of their paper, JO list several versions of the JO statistic. In this
paper, we employ Equation (10) (see Page 3 of JO), which is the log-version of their test-statistic, and it is shown below:

\[
\frac{\sum_{i=1}^{N} \left( \ln S_{w,V_i} - \ln RV_{i} \right)}{\sqrt{\Omega_{S_{w}}} \Sigma} \xrightarrow{d} \mathcal{N}(0,1)
\]

where \( \Sigma \) is a well-defined strictly positive càdlàg semimartingale process of locally bounded variation with \( \int_0^T \sum_i dt < +\infty, \forall T > 0 \) and \( S_{w,V_i} \) is shown in their Equation (4) as follows:

\[
S_{w,V_i} = 2 \sum_{i=1}^{N} (R_i - \mu_i)
\]

and \( RV_i \) is shown in their Equation (6) as follows:

\[
RV_i = \sum_{i=1}^{N} \mu_i^2
\]

where \( R_i = \frac{S_{i \infty}}{S_{(i-1) \infty}} - 1 \) and \( \mu_i = \ln \frac{S_{i \infty}}{S_{(i-1) \infty}} \). Put differently, \( R_i \) is the simple return and \( \mu_i \) is the continuously compounded return or log return. Finally, 

\[
\Omega_{S_{w,V}} = \frac{1}{n} \mu_i \int_0^T \sum_i \Delta_i dt
\]

and \( \mu_b = E(\chi^4) \) for \( \chi \sim \mathcal{N}(0,1) \) and these definitions are provided on page 3 of JO.

Using the JO measure, the correlations of jumps are 0.07 on a monthly basis and -0.06 on a semimonthly basis, which support the previously reported co-movement of jumps detected by the LM statistics. Estimations of the co-dependence in jumps corresponding to the LM measure and the correlations in jumps corresponding to the JO measure lead us to conclude that jumps
between the two series are not significantly correlated. However, in Section 4, we demonstrate that they are highly correlated around important economic events.

4. JUMPS, CONTINUOUSLY COMPOUNDED RETURNS OF INTEREST RATES, AND FOMC AND MPC MEETINGS

In this section, we examine whether or not jumps in the two series of continuously compounded returns of T-bill and Gilt rates are correlated around major economic events. To accomplish this task, we investigate whether the periods of highest jumps of both returns of US and UK interest rates occur at the same time as the FOMC and MPC meetings. We implement the Anderson et al. (2010) method to enable a comparison of the JO measure and the LM measure. Absent the sequential jump adjustment procedure proposed by Anderson et al. (2010), only the LM measure can be directly implemented using daily data. Estimating jumps on a daily basis allows us to identify periods with the highest jumps and show whether they occur before (lead) or after (lag) the FOMC meeting dates and MPC meeting dates. The FOMC meeting dates include one-day and two-day meetings, conference calls, and unscheduled meetings and the MPC meeting dates include two-day meetings and unscheduled meetings.

We begin with the implementation of the LM measure. Table 3 reports the top 1% of estimated LM statistics in absolute value as these correspond to the days with the highest LM statistics (i.e. the highest jumps) in absolute value. Panel A of Table 3 displays LM statistics estimated from the T-bill data, while Panel B reports estimates based upon Gilt yields data. The results in Panel A of Table 3 indicate that 57% of FOMC unscheduled meetings (8 out of 14
unscheduled meetings) were held when the jumps in the continuously compounded returns of T-bill rates were the highest and most of them (7 out of 8) lagged the jumps. So, our results indicate that FOMC news releases lagged jumps in the continuously compounded returns of US interest rates and this implies the US T-bill reacted before the announcement. Our findings here speak to the efficiency in the US market in regards to the arrival of information contained in FOMC news releases. As expected, since 1-day and 2-day meeting dates are well known to the public in advance, they also lagged jumps in T-bills. We also observe that the top five highest positive (negative) jumps occurred on August 15, 2007, July 30, 2013, April 30, 2013, April 2, 2013, June 15, 2012 (June 2, 2009, December 4, 2007; March 2, October 7, and March 24, 2010), respectively. The first top two negative jumps are also during the NBER recessions. The other three are in 2010, which is during the subprime mortgage crisis. Three of the top positive jumps are in 2013 during the ongoing European debt crisis. Jumps in T-bills led MPC 2-day meetings 12 times and lagged them 13 times, thus implying jumps in the US T-bill follow no lead/lag pattern with MPC 2-day meetings. This is expected since 2-day meetings in the UK are known to be held during the first ten days of every month. Consistent with the aforementioned findings regarding jumps in T-bills and US unscheduled meetings, jumps in T-bills also led all three MPC unscheduled meetings. Overall, the evidence supports the idea that US T-bills react to news information contained in FOMC and MPC unanticipated events in advance.

The results displayed in Panel B of Table 3 display jumps in Gilt returns. One-third of negative jumps in Gilt returns occurred during the NBER recession periods. 57% of FOMC unscheduled meetings (8 out of 14) occurred during the highest jumps in the returns of UK
interest rates. Interestingly, most of the jumps in Gilt returns (7 out of 8) lagged FOMC and MPC unscheduled meetings in contrast with jumps in the returns of US interest rates leading both FOMC and MPC unscheduled meetings. Jumps in Gilt returns led 1-day and 2-day meetings (15 out of 19 days), which is not surprising since FOMC 1-day and 2-day meeting dates are posted on the Fed website in advance. FOMC news releases led jumps in the returns of UK interest rates. This implies that UK Gilts react to the FOMC announcement and speak to how information in it gets impounded in UK markets. The jumps in Gilts seem to be more responsive to the FOMC meetings than the MPC meetings.

*** Table 3 ***

The results from the implementation of the JO measure are displayed in Table 4 which reports the top 1% of estimated JO statistics in absolute value as these correspond to the days with the highest JO statistics (i.e., the highest jumps) in absolute value. Panel A of Table 4 displays JO statistics estimated from the T-bill data, while Panel B reports estimates of the JO statistics based upon Gilts data. The results in Panel A imply that 71.43% of FOMC unscheduled meetings (10 out of 14 unscheduled meetings) were held when jumps in continuously compounded returns of T-bills were the highest and most of them (8 out of 10) lagged jumps. Jumps in T-bill returns also led all three MPC unscheduled meetings. Jumps in the US returns also led two out of three FOMC conference calls and all two 2-day meetings, but lagged three out of four of the 1-day meetings. Similar to unscheduled meetings, conference calls are not scheduled in advance. The results under the JO measure are consistent with those under the LM measure. Jumps in T-bill returns led unanticipated FOMC and MPC news release. Astonishingly,
all of the negative jumps occurred during the NBER recession periods and one-third of the positive jumps occurred in 2010.

The results corresponding to the implementation of the JO measure using Gilts data, which is displayed in Panel B show that 50% of the top negative jumps in Gilt returns happened during NBER recessionary periods. Jumps occurred during 13 out of 14 unscheduled FOMC meetings, 8 of which led jumps. Thus, FOMC news releases led jumps in the returns of UK interest rates. In contrast to the results from the LM measure, jumps in Gilt returns led all three MPC unscheduled meetings. With conflicting results between JO and LM measure for Gilts, we hesitate to draw any conclusion about the pattern of jumps in UK gilts to MPC unanticipated events.

***Table 4***

Implementation of both the JO measure and the LM measure leads us to conclude that the Fed’s FOMC meetings, especially unscheduled ones and conference calls, mostly lagged the highest jumps of T-bill returns, but led those of Gilt returns. They lagged jumps in US T-bill returns by 4 days and 4.3 days (including non-working days), based upon the LM and JO measures, respectively. According to the LM and JO measures, FOMC unscheduled meetings and conference calls led jumps in Gilt returns by 3.7 days and 3 days, respectively.

4.1 Robustness Check

4.1.1 Influential Observations of Jump Regressions on Time Trend
To demonstrate the robustness of the findings, we detect the days that have unusual jumps by computing influence statistics, which is a method of discovering the influential observations (days in our study) or the outliers of jump regressions on a time trend. They measure the difference that a single observation makes to the regression of jump statistics (in our case LM statistics and JO statistics) on a time trend. Out of 38 influential dates of jumps in T-bill returns, 57% of the Fed meetings lagged, 35% led, and 8% were on the same date. Only 6.1% of the MPC meetings occurred during the most influential dates but there is no lead/lag pattern between them. The results demonstrate that meetings by the Fed lagged the most influential period of the 3-month T-bill returns and are consistent with our finding that meetings by the Fed lagged jumps in the returns of US interest rates.

Additionally, our analysis of the Gilt data indicates that 65% of FOMC unscheduled meetings occurred within one week of influential dates in Gilts and 32% of conference calls were held by the Fed during Gilt influential periods. Out of 38 influential dates of jumps in Gilts, 49% of the FOMC unscheduled meetings and conference calls led, 40% lagged, and 11% were on the same date. The evidence suggests the most influential period of Gilts occurred after the FOMC unanticipated meetings, which is in contrast to the results of the US T-bill. There was no clear lead/lag pattern between MPC meetings and influential periods of Gilts.

Overall, there were more meetings held by the Fed than by the Bank of England during the influential periods in Gilts. Most of the highest jump days and the days with dramatic changes in T-bill returns occurred before the Fed meetings while changes in Gilt returns occurred after them, the majority of which were unscheduled meetings.
4.1.2 General Robustness Check

We also apply our jump measures to US LIBOR data as well as UK LIBOR data corresponding to the three-month maturity and the results remained qualitatively intact. This demonstrates robustness of our findings. T-bills and Gilts are likely to jump due to changes in the market expectations in interest rates, whereas, jumps may be induced in LIBOR rates by credit risk changes as well as changes in interest rate risk. For example, T-bill rates and Gilt rates are likely to jump when unexpected monetary action changes market expectations for interest rates, while LIBOR can also jump due to sudden changes in credit risk in the market.

5. JUMP CORRELATIONS AND ECONOMIC EVENTS

In this section, we examine the periods contributing the most to the jump correlation across the returns of US and UK interest rates and whether these periods contain important economic events. To accomplish this task, we identify the most influential period or the period that contributes the most to correlated jumps. Then, we examine whether there is a lead/lag effect between the influential periods and FOMC and MPC meetings. Finally, we connect FOMC meetings and MPC meetings to major economic events. For the LM measure, we do not compute the correlation because the numerator of the LM measure is return and we do not want to contaminate the correlation of the LM measure by return correlation. To estimate the jump correlation using the LM measure, we would need to apply a non-parametric measure involving Chi-square statistics, which does not allow us to identify the influential period. To identify the
most influential period and relate it to macroeconomic events, we apply the JO measure and find that, unlike the influential periods corresponding to the raw returns of interest rates, which are displayed in Table 2, September 2001, October 2008, and January 2009 are the most influential periods. September 2001 is the period when the Fed made two conference calls and the Bank of England had an unscheduled meeting. January 2009 is the period when the Fed had an unscheduled meeting. Both time periods are during the recession periods defined by NBER. The second half of October 1998 is when the Fed had a conference call.

Overall, jumps significantly exist in both T-bill and Gilt returns and although jumps between the two series are not significantly correlated, the most influential periods of correlated jumps include FOMC conference calls and MPC unscheduled meetings. This means that the period, which most strongly influences the correlation in jumps in the returns of US and UK interest rates include unexpected meetings by both the Bank of England and the FOMC. Two of the eleven conference calls were held on September 13 2001 and September 17 2001 and one of the three MPC unscheduled meeting was September 18 2001. All meetings followed the terrorist attacks on the US, which occurred on September 11th 2001. Brounen and Derwall (2010) note that, of the economic events they studied, the terrorist attacks of September 11 were the only event, which led to long-term effects on international financial markets. Moreover, while Johnston and Nedelescu (2006) note the global impact of the September 11 terrorist attacks on financial markets, they also remark that due to spillover effects, the impact was even greater for Europe after September 17, 2001. Additionally, the impact of some terrorist attacks occurring on European soil (e.g., the Madrid Bombings occurring in 2004) had a less profound impact on
European financial markets than the September 11 attacks taking place in the US (Johnston and Nedelescu 2006).

Additionally, our results also imply that October 1998 and September 2009 contribute very strongly to the correlation in jumps across the US and UK during which many important international financial market events took place (e.g., Russian Default Crisis in Autumn 1998 and Brazilian Currency Crisis in 1998 and Global Financial Crisis in 2009). See BIS Committee on the Global Financial System (1999) for more information. Taken together, these results suggest that dependencies across the US and UK financial markets increase during crises times and are consistent with findings of Gorton (2009) and Grundke and Polle (2012).

6. CONCLUSION

The focus of this paper has been to investigate monetary policy behavior across the US and UK over the period 1997-2013 through the exploitation of the relationship between jumps in interest rates, and macroeconomic news releases related to monetary policy. We detect jumps using the LM measure and JO measure. Although the LM measure is the only measure that enables us to detect jumps in daily data, the sequential jump adjustment method of Anderson et al. (2010) allows us to provide robustness to our implementation of the LM measure through the JO measure. We limit the scope of our study to the US and UK due to a long history of cooperation across these two countries and choose to study the period 1997-2013 because of the many important economic events faced by policy makers. As there have been many news announcements that have been shown to be significant in the extant literature, we focus only on
those announcements that are related to monetary policy; these are broadly linked to FOMC/MPC meetings.

We conclude that FOMC unscheduled meetings and conference calls lag jumps of continuously compounded returns to T-bill yields, but lead jumps of continuously compounded returns to Gilt yields. Jumps across these two series are highly correlated during important economic events. Over our sample period, estimation of the LM measure implies that the FOMC unscheduled meetings and conference calls lagged jumps in US T-bills and led jumps in Gilts by 4 days and 3.7 days, respectively, while estimation of the JO measure enables us to conclude that FOMC unscheduled meetings and conference calls lagged jumps in US T-bills and led jumps in Gilts by 4.3 days and 3 days, respectively. Our findings are important because they suggest that around important economic events, jumps in the returns of US and UK interest rates are strongly correlated. Our study reveals such events to include the terrorist attacks on US soil occurring on September 11, 2001, the Brazilian Currency Crisis, the Russian Default Crisis, and the Global Financial Crisis.

Additionally, our results speak to the importance of FOMC unscheduled meetings and conference calls as important predictors for jumps in the returns of UK and US interest rates. To provide some theoretical context to this finding, we also provide results from the maximum likelihood estimation of the Poisson-Gaussian continuous-time model developed by Das (2002), which indicate that FOMC unscheduled meetings and conference calls are statistically significant predictors of jumps in US T-bill data and UK gilt data. Of the several theoretical models proposed in the extant literature, we select the continuous-time Poisson-Gaussian model of Das
(2002) because it provides us with a simple and tractable theoretical framework in which to link interest rates to jumps and jumps to FOMC/MPC meetings.

Our empirical analysis identifies some potentially fruitful avenues for future work. Specifically, the fact that dependencies across US and UK financial markets increase during crises only exacerbates concerns regarding the impact of such economic events on broader global financial markets. One interesting extension to our study includes an examination of how dependencies across North American markets and European markets change during crisis periods. A second potentially worthwhile extension to the current research involves an econometric examination of possible spurious correlation between times of FOMC/MPC meetings and times of jumps.17 This extension is motivated by Huang and Tauchen (2005) who use Monte Carlo simulation analysis to conclude that microstructure noise biases the tests against the detection of jumps. Such market microstructure noise can spuriously induce jumps.

Appendix A: The Sequential Jump Adjustment Procedure of Anderson et al. (2010)

The variance swap test of Jiang and Oomen (2008, henceforth JO) cannot be used to detect jumps over a daily interval. Anderson et al. (2010) propose a sequence of steps, which can be employed to overcome this problem and identify jumps in daily returns. This jump adjustment enables us to compare the findings of LM and JO measures.

This appendix provides more detail on the sequential jump adjustment of Anderson et al. (2010). Before delving into the details of the sequential jump adjustment procedure, we provide a
brief overview of some pertinent variable definitions. We define the realized volatility on day \( t \) as

\[
RV_t = \sum_{i=1}^{M} r_{i,j}^2 \quad t = 1, \ldots, T
\]

(A.1)

In Equation (A.1), \( r_{i,j} \) is the \( j^{th} \) intra-day return on day \( t \). \( T \) is the number of trading days and \( M \) is a variable that determines the precision of the realized volatility estimate and jump measure. As \( M \to \infty \),

\[
RV_t \to_p \int_{t-1}^{t} \sigma^2(s)ds + \sum_{q=m-1}^{q_{i,j}+1} \kappa^2(s) \quad t = 1, \ldots, T
\]

(A.2)

In Equation (A.2), \( q(t) \) is a counting process and \( \kappa \) is the jump size. Barndorff-Nielsen and Shephard (2004, 2006) show that the right hand side of Equation (A.2) can be defined in terms of the bi-power variation measure as follows

\[
BV_t = \mu_2^{-2} \sum_{j=2}^{M} |r_{i,j}| |r_{i,j-1}| \quad t = 1, \ldots, T
\]

(A.3)

In Equation (A.3) above, \( \mu_1 = \sqrt{\frac{T}{\pi}} \) and as \( M \to \infty \),

\[
BV_t \to_p \int_{t-1}^{t} \sigma^2(s)ds \quad t = 1, \ldots, T
\]

The jump component would consist of the difference between the realized volatility and the bi-power variation component, and so Anderson et al. (2010) assess the significance of the daily jump component using the following test statistic:

\[
Z_t = \sqrt{M} \frac{\ln RV_t - \ln BV_t}{\left( \mu_2^{-2} + \mu_1^{-2} \right)^{1/2}} \to_d N(0,1)
\]

(A.4)
In Equation (A.4), TQ is the realized tripower quarticity measure and it is defined as

\[ TQ_t = \frac{1}{\sqrt{\pi}} \mu t \frac{1}{2} \sum_{i=1}^{T} \left[ r_{t,i} \left| r_{t,i-1} \right| r_{t,i-2} \right]^{1/3} t = 1, \ldots, T \tag{A.5} \]

In Equation (A.5), \( \mu t \frac{1}{2} = 2^{1/3} \Gamma \left( \frac{7}{6} \right) \Gamma \left( \frac{1}{2} \right) \) where \( \Gamma \) is the gamma function. These equations (A.1-A.5) will enable us to identify a sequence of steps to facilitate the identification of jumps in daily returns. The intuition behind the sequence of steps is as follows. We are interested in the possible detection of several jumps. In identifying the first jump, the realized variation is computed based upon Equation (A.1). If the test statistic in (A.4) rejects, then there is at least one jump on day \( t \). Its contribution is the difference between the largest squared intraday return and the remaining (M-1) intra-day returns. For the identification of the possible second jump, we would define the realized volatility corrected for one jump in terms of the sum of the squared returns where the first jump is replaced by the remaining intra-day returns. If the new statistic, which is given by Equation (A.4) with \( RV_t \) replaced with the new corrected realized volatility still rejects, then we conclude that there are at least two jumps and we continue the procedure until the test in Equation (A.4) no longer rejects the null. If we fail to reject the test, then the sequential testing procedure stops.

**Appendix B: Model-free Measures and the Poisson-Gaussian Continuous-time Model of the Short-term Interest Rate: Theoretical Model Framework for our Findings**

The purpose of this appendix is to confirm behavior patterns of the FOMC and MPC revealed in Sections 4 and 5 using the Poisson-Gaussian model.
There are several models that have been developed to study interest rates in the presence of jumps (see, e.g., Ahn and Thompson 1988; Duffie, Pan, and Singleton 2000; Das 2002; Piazzesi 2005; Heidari and Wu 2010). We select the model of Das (2002) because it provides us with a parsimonious and tractable theoretical framework in which to link interest rates to non-parametric jump measures as well as link jumps to market events and FOMC/MPC meetings. Before we provide the results from estimation of the Poisson-Gaussian model using the data described in the body of the paper, we provide an analytical treatment of the theoretical development of the model.

Accordingly, and mostly adopting the notation of Das (2002), we assume jumps in interest rates arise from the actions of the Fed. Following Baz and Das (1996), Das (2002), or Johannes (2004), the basic process for the short-term interest rate is shown below:

\[ dr = \kappa (\theta - r) dt + \sigma d\omega + J d\pi(h) \]

where \( \theta \) is the long-run mean for \( r \), which is a short-term interest rate that mean reverts at rate \( \kappa \). \( \omega \) delineates the standard Weiner process with associated standard deviation, \( \sigma \). The mean reverting stochastic process also embodies a random jump component, which will be governed by a Poisson distribution \( \pi \) with arrival frequency \( h \). The jump size, which is given by \( J \), will be drawn from a normal distribution with mean \( \mu \) and variance, \( \gamma^2 \). This one-dimensional stochastic process for the short-term interest rate is also employed by Johannes (2004). From Das (2002), the discrete version of the model for the short-term rate is shown below:
\[ \Delta r = \kappa (\theta - r) \Delta t + \sigma \Delta \omega + (\mu + \gamma \Delta \omega) \Delta \pi(q) \]

where \(\Delta \omega\) is drawn from a normal distribution and \(\pi(q)\) is a Poisson process, which will be approximated by a Bernoulli distribution with parameter \(q\). The parameter \(q\) is the jump intensity and it controls the arrival rate of jumps. By using dummies to represent scheduled meetings as well as unscheduled meetings, we can assume that the jump arrival intensity is affected by scheduled meetings and unscheduled meetings as shown below:

\[
q(t) = \alpha_0 + \alpha_1 f_{1,t} + \alpha_2 f_{2,t} + \alpha_3 f_{3,t} + \alpha_4 f_{4,t}
\]

where \(f_{1,t}\) is a dummy variable that is one on the day of the one-day meeting and zero otherwise, \(f_{2,t}\) is a dummy variable that is one on the first day of a two-day meeting and zero otherwise, \(f_{3,t}\) is a dummy variable that is equal to one on the day of the unscheduled conference call and zero otherwise, and finally \(f_{4,t}\) is a dummy variable that is one on the day of an unscheduled meeting and zero otherwise.

Simplification of the discretized version of the model for short term interest rates leads us to the following equation which defines the conditional density function for the state variable, with \(s > t\),

\[
f(r(s)|r(t)) = q(t)N(r(t) + \kappa (\theta - r(t))\Delta t + \mu \sigma^2 \Delta t + y^2) \\
+ (1 - q(t))N(r(t) + \kappa (\theta - r(t))\Delta t, \sigma^2 \Delta t) \tag{B.1}
\]
The conditional density function displayed in Equation B.1 approximates the true Poisson density. To see this, we note that in the case that there is a jump, our conditional density function simplifies to

\[ q(t)N(t(r(t) + \kappa (\theta - r(t))\Delta t + \mu, \sigma^2 \Delta t + \gamma^2) \]

and when there is no jump, the conditional density simplifies to \((1 - q(t))N(t(r(t) + \kappa (\theta - r(t))\Delta t, \sigma^2 \Delta t)\). We define the indicator variable \( I_j = 1 \) if there is a jump and \( I_j = 0 \) if there is no jump. Then the indicator variable \( I_j \) is a Bernoulli trial and the variable \( B = \sum_{j=1}^{N} I_j \) is distributed as binomial. Thus, the probability of exactly \( n \) jumps out of the \( N \) days in the sample

\[
\Pr[B = n] = \binom{N}{n} q^{n}(1 - q)^{N-n}
\]

\[
= \frac{N!}{n!(N-n)!} q^{n}(1 - q)^{N-n}
\]

\[
= \frac{n!(N-1)! \cdots (N-n+1)! (n-n)!}{n!(N-n)!} q^{n}(1 - q)^{N-n}
\]

\[
= \frac{n!(N-1)! \cdots (N-n+1)}{n!} q^{n}(1 - q)^{N-n}
\]

Then, we let the sample size, \( N \), get large, and follow Das (2002) in defining \( q = h\Delta t + O(\Delta t) \) with \( \Delta t = T/N \), where \( T \) is the time between observations and \( h \) is a parameter that governs the frequency of arrival for jumps. These definitions imply that the probability of observing exactly \( n \) jumps in our \( N \) sampled days is given by,
Moving from step 2 to step 3 in the simplification above, we note that
\[\lim_{N \to \infty} \frac{\kappa_1 \cdots \kappa_{n-1} \cdots \kappa_{n+1}}{N!} \left( 1 - \frac{\kappa_1}{N} \right)^N = 1\]
and
\[\lim_{N \to \infty} \left( 1 - \frac{\kappa_1}{N} \right)^{-N} = (1)^{-n} = 1\]
as \(N\) gets large. Finally, as \(\lim_{N \to \infty} \left( 1 - \frac{\kappa_1}{N} \right)^N\) is defined to be \(e^{-\kappa_1}\), the probability of observing exactly \(n\) jumps in \(N\) sampled days is given by

\[\Pr[B = n] = \frac{\kappa_1 \cdots \kappa_{n-1} \cdots \kappa_{n+1}}{n!} e^{-\kappa_1}\]

This is the true Poisson density function with parameter \(hT\). Thus, we have shown that the mixture of normal distributions provided in Equation B.1 approximates the true Poisson density and the Poisson distribution is commonly used in finance in the theoretical development of model frameworks employed to study jumps (see, e.g., Duffie and Singleton 1999; Glasserman and Kou 2003; Mayer, Schmid, and Weber, 2012).

As the conditional density function provided in Equation B.1 above, given by the mixture of normal distributions, approximates the true Poisson density, we proceed with parameter estimation which involves maximizing the log-likelihood function \(L\), where
which solves the optimization problem provided in Equation B.3 below

\[ L = \prod_{t=1}^{T} f(r(t + \Delta t)|r(t)) \]  

(B.2)

subject to: \( 0 \leq q \leq 1 \)

Please see Das (2002) for more information. The solution to the constrained optimization problem specified above for the US T-bill yield and UK Gilt yield data is displayed in Table B.1.

***Table B.1 about here***

The results are very similar across both countries’ interest rate data, providing robustness to the observed similarities mentioned in Section 3. It is clear from Table B.1 that both FOMC conference calls and unscheduled meetings increase the possibility of jumps in interest rates for the US and UK. \( \alpha_{\text{unschedmeet}} \) and \( \alpha_{\text{confcall}} \) are both significant at the 1% level for the maximum likelihood estimation of T-bill data and at the 5% and 1% level, respectively for that of Gilt yield rates. The coefficient for the 1-day FOMC meeting is also significant at the 5% level but its coefficient is much smaller than those associated with unscheduled meetings and conference calls in magnitude. The coefficient of 2-day US meetings is not significant for the T-bill data but negative and significant for the UK Gilt data. In regards to the UK Gilt data, the negative
coefficient associated with both the 1-day and 2-day FOMC meetings has to do with the fact that the target range for the Fed Funds rate has been 0-0.5% and it has not been changed by the Fed since December 2008. So, we interpret the lack of an anticipated change in the Federal Funds rate became a negative signal from the perspective of market participants in England. This reflects the closeness and cooperation that the Bank of England has with the Federal Reserve. Following similar logic, 1-day UK meetings are negative and significant for both datasets. The MPC has kept interest rates at a record low value of 0.5% since March 2009 and so for similar reasons as were described above for the US, we take the lack of change as a negative signal from the perspective of market participants in England and the US. Again, these results reflect the closeness and cooperation that the Bank of England has with the Federal Reserve. Moreover, the statistically significant coefficients are small in magnitude relative to the coefficients for unscheduled meetings and conference calls. The weak impact of the MPC scheduled meetings is expected since they are normally held during the first ten days in each month. Similarly, FOMC scheduled meetings are posted on the Fed website in advance. Implications of maximum likelihood estimation results using LIBOR USD and LIBOR GBP data are identical. For brevity, they are excluded from our final results, but are available upon request from the authors.

References


Januj A. Juneja is from the Department of Finance, College of Business Administration, San Diego State University, San Diego, California 92182. Email address: jjuneja@mail.sdsu.edu. Tel: (619) 594-8397, Fax: (619) 594-3272. Kuntara Pukthuanthong is from the Department of Finance, Trulaske College of Business, University of Missouri, Columbia, Missouri 65211. Email address: pukthuanthongk@missouri.edu. Tel: (573) 884-9785.

See Collin-Dufrense, Goldstein, and Jones (2008) for why model-independent or model-free structures may also provide estimation advantages in term structure estimation in the absence of jumps.

Initially, we included the ECB in our analysis, but we excluded it from our final results because Bean (1998) notes that the Bank of England sets interest rates to achieve targets for inflation and monetary policy is set according to this objective. In contrast, the ECB is compelled to maintain the value of the currency and this is more important than a precise objective of monetary policy. Based upon this information, the Bank of England sets monetary policy in a more similar manner to the Fed, rather than the ECB (see, e.g., de la Dehesa (2012) for more motivation that the Bank of England and the Fed conduct Monetary policy in a more similar fashion than the ECB).

Please see Appendix A for an overview of the sequential method.

These 136 meetings exclude the three unscheduled meetings which happened on the weekend or when markets were otherwise inactive.

Please see Jiang and Oomen (2008) or Lee and Mykland (2008) for more information. We thank both of the referees for this comment.

An overview of the Anderson et al. (2010) sequential procedure for jump detection is presented in Appendix A.

Due to limited space, we do not report these results but they are available upon request.

We thank the second referee for this point.

For the sake of convenience, we adopt the same notation as Anderson et al. (2010).

For convenience, we adopt the notation of Das (2002).

For simplicity, we drop the t argument, but this variable does depend upon t.