Do Affine Models Adequately Represent the International Asset Price Dynamics?

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Abstract

This paper examines whether affine models do an adequate job of representing the *joint* dynamics of international asset prices. We propose a simple arbitrage-based affine model in which stocks, bonds, and currencies of two countries are priced jointly. The specification is flexible enough to capture the stylized facts of international asset returns. We estimate the proposed affine model using the Efficient Method of Moments (EMM). Estimation results indicate that the proposed affine model fits the first and second moments well, but has difficulties fitting higher order moments. It also appears that the proposed model does a good job of representing the cross-asset dynamics. Finally, the proposed model does a better job fitting the joint dynamics of currencies and short rates than that of currencies and stocks.

I. Introduction

Affine models have been the workhorse model in the asset pricing literature for almost three decades. The celebrated Black and Scholes (1973) model falls into the affine class. Vasicek (1977) and Cox, Ingersoll, and Ross (1985) are the first generation of affine term structure models (ATSM). Duffie and Kan (1996) present a general framework for multi-factor ATSMs. In addition to stocks and bonds, affine models have also been widely used in modeling currencies and commodities.[1] Heston (1993) obtains a closed-form solution for the European option price in an affine stochastic volatility model. The key insight of Heston (1993) is that, although the conditional distribution of future asset prices is unknown, its characteristic function can be solved in closed form. This insight proves to be quite general. Bakshi and Madan (2000) and Duffie, Pan, and Singleton (2000) have recently established the tractability of affine models in pricing a large class of derivative assets.

Although affine models possess tractability superior to competing models,[2] they also impose several potentially restrictive assumptions. For example, affine models require that the drift terms and the squared diffusion terms of the stochastic processes be linear in state variables. If the model allows for random jumps, the jump intensity must be a linear function of state variables as well. Finally, affine models generally do not allow for a flexible correlation structure between factors (Dai and Singleton (2000)). How restrictive are these assumptions? How do affine models fare empirically? How many factors are needed? These questions have been the focus of numerous recent academic studies.

Existing tests of affine models have focused on the dynamics of term structure of interest rates, stock indexes, or exchange rates, usually in isolation. Most of these studies focus exclusively on the term structure of interest rates. Dai and Singleton (2000) explore the features of ATSMs that are important for explaining the joint distribution of yields on short- and long-term bonds. They show that extant affine models fail to describe the important features of the data and the source of the model misspecification is overly strong restrictions on the correlation among the state variables. Duffee (2002) investigates whether affine models of term structure of interest rates can forecast the future interest rate well. Duffee finds that standard affine models fail to provide a better forecast than the one produced by simply assuming that yields are martingales. He shows that the "essentially affine" models, which allow the compensation for interest rate risk to vary independently from interest rate volatility, produce more accurate forecasts of future yields.

Many other papers in this literature focus on the dynamics of stock indexes, currencies, or commodities. Andersen et al. (2002) estimate an affine jump diffusion model of stock index returns. Bakshi, Cao, and Chen (1997) and Bates (2000) test affine models using option prices. Additionally, Bates (1996) and Schwartz (1997) focus on currencies and commodities, respectively.

The common feature of the above studies is that they focus on a single asset class. In this paper, we extend the existing literature by examining multiple asset classes and focusing on cross-asset dynamics. Specifically, we investigate the performance of a simple affine model in capturing the joint dynamics of stock, bond, and currency markets of two countries. This extension is important because financial markets have become increasingly integrated and globalized. Developing and testing *integrated* asset pricing models are of critical importance to many practical finance applications that include assessing the value of international diversification, determining the optimal risk management strategies, and pricing cross-asset derivative securities.

The rest of the paper proceeds as follows: In Section II, we develop a simple two-country affine model that prices stock indexes, bonds, and currencies jointly. We demonstrate that this model is capable of capturing important features of the data. In Section III, we provide goodness-of-fit tests of the proposed affine model using the Efficient Method of Moments (EMM) of Gallant and Tauchen (1996). Strengths and weaknesses of the proposed model are discussed. Section IV concludes.

II. A Two-Country Affine Model

In this section, we develop a two-country model that is similar in structure to Nielsen and Saa-Requejo (1993), Ahn (1997), and Bakshi and Chen (1997a, 1997b). In the theoretical literature of international finance, there are basically three modeling approaches: (a) versions of international CAPM; (b) international general equilibrium models, and (c) arbitrage-based models in which the structure of the underlying economy needs not be explicitly specified. Our model falls in the third category.

A. The Setup

We consider an economy with two countries. Without loss of generality, one country is designated the domestic country and the other is designated the foreign country. Each country has its own currency. We also assume perfect mobility of capital and goods across the border. We fix a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and an information filtration $\{\mathcal{F}_t\}$ satisfying usual technical conditions (see, for example, Duffie (1996)). The probability measure $\mathcal{P}$ represents the common beliefs held by all the agents.

The stochastic environment is determined by the standard N-dimensional vector Brownian Motion $\omega = (\omega_1, \omega_2, ..., \omega_N)'$. These N sources of uncertainty are equivalently reflected by N economy-wide state variables $X = (X_1, X_2, ..., X_N)'$. X follows a vector diffusion process:

$$dX(t) = \mu_x(t, X)dt + \sigma_x(t, X)d\omega(t) \tag{1}$$

with $X(0) > 0$, where the drift $\mu_x(t, X)$ is an N-dimensional vector of expected instantaneous changes in $X(t)$ and the diffusion term $\sigma_x(t, X)$ is a full-rank $N \times N$ covariance matrix. Both the drift and diffusion terms are functions of t and the general state of the economy $X(t)$, and they satisfy the local Lipshitz and growth conditions, implying that there is an unique solution to the stochastic differential equations. Additionally, both $\mu_x(t, X)$ and $\sigma_x(t, X)\sigma_x(t, X)'$ are linear in $X(t)$. This is a necessary condition for an affine economy.

We assert the existence of a positive *nominal* pricing kernel in each of the two countries. The nominal pricing kernel for the domestic (foreign) country is denoted by $M(t)$ ($M^*(t)$). In a representative agent economy, the pricing kernel is proportional to the intertemporal marginal rate of substitution. It is well known that the absence of arbitrage implies the existence of a pricing kernel (see, for example, Duffie (1996)). We assume that markets are complete, which implies that the pricing kernel is unique for each country.

We assume that the time-series processes of the pricing kernels of the two countries are governed by the following two stochastic differential equations:

$$\frac{dM(t)}{M(t)} = \mu_m(X, t)dt + \sigma_m(X, t)d\omega \tag{2}$$

$$\frac{dM^*(t)}{M^*(t)} = \mu_m^*(X, t)dt + \sigma_m^*(X, t)d\omega \tag{3}$$

where $\mu_m(X, t)$, $\mu_m^*(X, t)$, $\sigma_m(X, t)\sigma_m(X, t)'$, and $\sigma_m^*(X, t)\sigma_m^*(X, t)'$ are linear in $X(t)$. The price at t of a domestic claim to a nominal payoff $F(T)$ at a future date T is then given by:

$$F(t) = E^{\mathcal{P}}\left[\frac{M(T)}{M(t)}F(T)|\mathcal{F}_t\right] \tag{4}$$

The risk premium faced by domestic agents is given by $-Cov_t\left(\frac{dF(t)}{F(t)}, \frac{dM(t)}{M(t)}\right)$. That is, the investors are willing to accept a lower return on an asset if its payoff tends to be high when the marginal utility is high.

The price of a discount bond that pays one unit of domestic currency at $t+\tau$ is given by:

$$B(t,\tau) = E^{\mathcal{P}}\left[\frac{M(t+\tau)}{M(t)}|\mathcal{F}_t\right] \tag{5}$$

Similarly for the foreign bond,

$$B^*(t,\tau) = E^{\mathcal{P}}\left[\frac{M^*(t+\tau)}{M^*(t)}|\mathcal{F}_t\right] \tag{6}$$

The short rate is the negative drift of the proportional pricing kernel process.

$$R(t) = -\mu_m(t,X) \tag{7}$$

$$R^*(t) = -\mu_m^*(t,X) \tag{8}$$

We define the exchange rate S as the price of foreign currency in terms of the domestic currency. Ahn (1997) shows that:

$$\frac{dS}{S} = \frac{d\left(\frac{M^*}{M}\right)}{\left(\frac{M^*}{M}\right)} \tag{9}$$

Thus, the exchange rate is proportional to the ratio of the foreign pricing kernel and the domestic pricing kernel.

To derive the pricing formula for the domestic market portfolio, we need to know the stochastic process for the aggregate dividend $D(X,t)$. Given $D(X,t)$, it follows that the price of the domestic market portfolio is:

$$P(t) = \int_t^\infty E_t\left[\frac{M(\tau)}{M(t)}D(\tau)\right]d\tau \tag{10}$$

The price of the foreign market portfolio is similar.

It is important to note that the above economy can be supported in a general equilibrium similar to Lucas (1982) and Bakshi and Chen (1997b). Specifically, one can model the money supply processes instead of the pricing kernels. With a "cash in advance" constraint and additional assumptions on preferences, one can establish a one-to-one relation between the money supply and the pricing kernel.

B. A Specific Example

In this subsection, we describe an explicit affine model that is the subject of our empirical analysis. The factor structure of this model is similar to Ahn (1997). There are three state variables, each following a square-root process.

$$dX(t) = \kappa_x(\theta_x - X(t))dt + \sigma_x\sqrt{X(t)}d\omega_1(t) \tag{11}$$

$$dY(t) = \kappa_y(\theta_y - Y(t))dt + \sigma_y\sqrt{Y(t)}d\omega_2(t) \tag{12}$$

$$dZ(t) = \kappa_z(\theta_z - Z(t))dt + \sigma_z\sqrt{Z(t)}d\omega_3(t) \tag{13}$$

with $d\omega_1$, $d\omega_2$ and $d\omega_3$ orthogonal to each other. In this setup, the conditional and unconditional correlations between X, Y, and Z are zero.

The drift of the domestic pricing kernel $M(t)$ is driven by state variables $X(t)$ and $Y(t)$ whereas the drift of the foreign pricing kernel $M^*(t)$ is driven by $X(t)$ and $Z(t)$. As such, $X(t)$ can be interpreted as the common factor and $Y(t)$ and $Z(t)$ are local factors. The diffusion terms of the domestic pricing kernel and foreign pricing kernel, however, are both driven by all three state variables $X(t)$, $Y(t)$, and $Z(t)$.

$$\frac{dM(t)}{M(t)} = -[\mu_m + \eta_1 X(t) + \eta_2 Y(t)]dt + \sigma_1 \sqrt{X(t)} d\omega_1(t) + \sigma_2 \sqrt{Y(t)} d\omega_2(t) + \sigma_3 \sqrt{Z(t)} d\omega_3(t) \quad (14)$$

$$\frac{dM^*(t)}{M^*(t)} = -[\mu_m^* + \eta_1^* X(t) + \eta_2^* Z(t)]dt + \sigma_1^* \sqrt{X(t)} d\omega_1(t) + \sigma_2^* \sqrt{Y(t)} d\omega_2(t) + \sigma_3^* \sqrt{Z(t)} d\omega_3(t) \quad (15)$$

The market portfolio is a claim to the aggregate output stream of the economy. We assume the domestic aggregate dividend $D(t)$ and foreign aggregate dividend $D^*(t)$ follow:

$$D(t) = (\Gamma_0 + \Gamma_1 X + \Gamma_2 Y) e^{\gamma_0 + \gamma_1 X + \gamma_2 Y} \quad (16)$$

$$D^*(t) = (\Gamma_0^* + \Gamma_1^* X + \Gamma_2^* Z) e^{\gamma_0^* + \gamma_1^* X + \gamma_2^* Z} \quad (17)$$

B.1. Discussion of the Model

Several remarks about this model are in order at the outset. First, our model is not maximally flexible as defined in Dai and Singleton (2000) for two reasons: (a) each country is driven by one common factor and one local factor, rather than by three common factors; (b) our model assumes zero *unconditional* correlations between three factors whereas Dai and Singleton show that positive unconditional correlations are also admissible. We feel that these two constraints are not particularly restrictive in our setting. Ahn (1997) argues that the distinction between common factors and local factors is important for generating flexible correlations. Moreover, unreported empirical results appear to show that our restricted model renders easier parameter identification than the maximal model, presumably because in the restricted model each factor has a distinct impact on the dynamics of asset returns. Allowing for nonzero unconditional correlations between state variables will certainly help fit correlations between different asset classes, but empirical evidence shown in Section III.C suggests that our model has little trouble fitting these correlations.

Second, Dai and Singleton (2000) contend that allowing for negative *conditional* correlations between state variables is important for explaining the dynamics of term structure of interest rates. To allow for negative conditional correlations, one or more of the factors have to be Gaussian, which would reduce the number of volatility factors (compared to the square-root model). We argue that multiple volatility factors are essential to approximate the long memory of asset return volatilities. Moreover, interest rates can become negative when there are Gaussian factors. We also note that Dai and Singleton focus on the joint distribution of the short-term and long-term interest rates whereas we focus on the joint dynamics of exchange rates, short rates, and stock indexes. It is not clear to what extent this particular result of Dai and Singleton can be generalized to our setting.

Finally, $X(t)$, $Y(t)$ and $Z(t)$ are latent state variables that are not readily interpretable. However, as will be seen in Sections II.B.2 through II.B.4, domestic short rates, foreign short rates, short rate volatilities, stock return volatilities and currency volatilities are all linear functions of state variables. One can easily transform $X(t)$, $Y(t)$, and $Z(t)$ to a set of new state variables that can be interpreted as the domestic short rate factor, foreign short rate factor, volatility factor, and so on. The transformed model would be functionally equivalent to the proposed model as demonstrated by Dai and Singleton (2000).

B.2. Bond Prices and Short Rates

Let $B(t,\tau)$ be the price of a domestic bond that pays one unit of domestic currency at $t+\tau$.

It can be shown that $B(t,\tau)$ (see Bakshi and Chen (1997a)) is:

$$B(t,\tau) = \exp[-\mu_m \tau - \alpha_x(\tau) - \alpha_y(\tau) - \varrho_x(\tau)X(t) - \varrho_y(\tau)Y(t)] \tag{18}$$

where,

$$\begin{aligned}
\varrho_x(\tau) &= \frac{2\eta_1(1-e^{\upsilon_x\tau})}{2\upsilon_x + (\kappa_x + \sigma_1\sigma_x - \upsilon_x)(1-e^{\upsilon_x\tau})} \\
\varrho_y(\tau) &= \frac{2\eta_2(1-e^{\upsilon_y\tau})}{2\upsilon_y + (\kappa_y + \sigma_2\sigma_y - \upsilon_y)(1-e^{\upsilon_y\tau})} \\
\alpha_x(\tau) &= \frac{2\theta_x}{\sigma_x^2}\left\{\ln\left[1 + \frac{(1-e^{-\upsilon_x\tau})(\kappa_x + \sigma_1\sigma_x - \upsilon_x)}{2\upsilon_x}\right] + \frac{1}{2}[\upsilon_x - \kappa_x - \sigma_1\sigma_x]\tau\right\} \\
\alpha_y(\tau) &= \frac{2\theta_y}{\sigma_y^2}\left\{\ln\left[1 + \frac{(1-e^{-\upsilon_y\tau})(\kappa_y + \sigma_2\sigma_y - \upsilon_y)}{2\upsilon_y}\right] + \frac{1}{2}[\upsilon_y - \kappa_y - \sigma_2\sigma_y]\tau\right\} \\
\upsilon_x &= \sqrt{(\kappa_x + \sigma_1\sigma_x)^2 + 2\sigma_x^2\eta_1} \\
\upsilon_y &= \sqrt{(\kappa_y + \sigma_2\sigma_y)^2 + 2\sigma_y^2\eta_2}
\end{aligned} \tag{19}$$

Bond yields are given by $\frac{1}{\tau}(\mu_m\tau + \alpha_x(\tau) + \alpha_y(\tau) + \varrho_x(\tau)X(t) + \varrho_y(\tau)Y(t))$. Clearly, bond yields are linear in state variables. This is a defining feature of all ATSMs (see Duffie and Kan (1996)). The corresponding instantaneous interest rate can be solved by taking the limit of the bond yield as τ goes to zero. More directly, using equations (7) and (8), the domestic instantaneous interest rate $R(t)$ is:

$$R(t) = \mu_m + \eta_1 X(t) + \eta_2 Y(t) \tag{20}$$

Similarly, we can show that the foreign instantaneous interest rate $R^*(t)$ is:

$$R^*(t) = \mu_m^* + \eta_1^* X(t) + \eta_2^* Z(t) \tag{21}$$

To ensure that $R(t)$ and $R^*(t)$ are always positive, all coefficients in equations (20) and (21) are assumed to be positive. The above short rate model captures several well-known stylized facts. First, both $R(t)$ and $R^*(t)$ are persistent and mean-reverting because $X(t)$, $Y(t)$ and $Z(t)$ are persistent and mean-reverting. Second, the volatility of $R(t)$ is $V_R(t) = {\eta_1}^2\sigma_x^2 X(t) + \eta_2^2\sigma_y^2 Y(t)$. The volatility of $R^*(t)$ is $V_{R^*}(t) = {\eta_1^*}^2\sigma_x^2 X(t) + {\eta_2^*}^2\sigma_z^2 Z(t)$. These volatilities are time-varying, persistent, and mean-reverting. Furthermore, because both the short rate and the volatility of the short rate are increasing functions of state variables X, Y, and Z, this model is capable of capturing the "level effect," which states that the interest rate volatility is higher when interest rates are higher.

B.3. The Exchange Rate

Using equations (9), (14), (15) and applying Ito's lemma, we obtain the following exchange rate dynamics:

$$\frac{dS(t)}{S(t)} = \{R(t) - R^*(t) + \sigma_1(\sigma_1 - \sigma_1^*)X(t) + \sigma_2(\sigma_2 - \sigma_2^*)Y(t) + \sigma_3(\sigma_3 - \sigma_3^*)Z(t)\}dt$$
$$+ (\sigma_1 - \sigma_1^*)\sqrt{X(t)}d\omega_1(t) + (\sigma_2 - \sigma_2^*)\sqrt{Y(t)}d\omega_2(t) + (\sigma_3 - \sigma_3^*)\sqrt{Z(t)}d\omega_3(t) \quad (22)$$

The uncovered interest rate parity clearly does not hold. The expected percentage change of the exchange rate is equal to the interest rate differential plus a risk premium. The exchange rate volatility $V_s(t)$ is $(\sigma_1 - \sigma_1^*)^2X(t) + (\sigma_2 - \sigma_2^*)^2Y(t) + (\sigma_3 - \sigma_3^*)^2Z(t)$. This volatility is stochastic, persistent, and mean-reverting, all of which are desirable features. In addition, $Cov\left(\frac{dS}{S}, dV_s\right) = \sigma_x(\sigma_1 - \sigma_1^*)^3X(t) + \sigma_y(\sigma_2 - \sigma_2^*)^3Y(t) + \sigma_z(\sigma_3 - \sigma_3^*)^3Z(t)$. This endogenously determined covariance can be positive or negative. Bates (1996) and many others have shown that this correlation is important for explaining the "volatility smile" in currency options.

As shown in Ahn (1997), the above specification of the exchange rate and interest rates can potentially account for the "forward premium puzzle". Backus, Foresi and Telmer (2001), however, argues that affine mdoels cannot address the puzzle. Unfortunately, this paper will not be able to provide additional empirical evidence on this issue because we use the first difference of interest rates (as opposed to the level) in our empirical analysis. By using the first difference, the link between exchange rate changes and the interest rate differentials cannot be preserved. Moreover, we use daily data in our empirical investigation. The forward premium puzzle, however, has been largely documented using data at much lower frenquency.

B.4. Stock Indexes

Using the Feynman-Kac formula, the price of the domestic market portfolio $P(t)$ satisfies the following partial differential equation:

$$\mathcal{D}P(t) - R(t)P(t) + D(t) = 0 \tag{23}$$

where $\mathcal{D}$ is the differential operator and $R(t)$ is the instantaneous interest rate given in (20). Solving the above PDE using the separation of variables technique, or using equations (10), (16) and Bakshi and Chen (1997a, equation (40)), we arrive at the following formula for the domestic stock index:

$$P(t) = Ae^{\gamma_1 X + \gamma_2 Y} \tag{24}$$

and foreign stock index:

$$P^*(t) = A^* e^{\gamma_1^* X + \gamma_2^* Z} \tag{25}$$

where A and A^* are complicated functions of the structural parameters.[3] A and A^* only affect the level of stock indexes and do not affect stock index returns, which are given below.

$$d \ln P(t) = \gamma_1 dX + \gamma_2 dY \tag{26}$$

$$d \ln P^*(t) = \gamma_1^* dX + \gamma_2^* dZ \tag{27}$$

The volatility of the domestic stock index return $V_p(t)$ is $\gamma_1^2\sigma_x^2X(t)+\gamma_2^2\sigma_y^2Y(t)$. The volatility of the foreign stock index return $V_p^*(t)$ is $\gamma_1^*\sigma_x{}^2X(t) + \gamma_2^*\sigma_z^2Z(t)$. Each of these volatilities follow a two-factor process. This specification is consistent with the evidence presented in Bates (2000), who calls for two volatility factors. Recent evidence suggests that stock return volatilities have a long memory. Although it is generally not possible to incorporate long memory directly into the volatility process without sacrificing the tractability of the model, a two-factor volatility model can provide a better approximation to a long memory process than a single-factor volatility model.[4]

$$cov\left(d\ln P(t), dV_p(t)\right) = \gamma_1^3\sigma_x^4X(t) + \gamma_2^3\sigma_y^4Y(t) \tag{28}$$

$$cov\left(d\ln P^*(t), dV_p^*(t)\right) = \gamma_1^{*3}\sigma_x^4X(t) + \gamma_2^{*3}\sigma_z^4Z(t) \tag{29}$$

It can be seen from equations (28) and (29) that the covariance between stock returns and volatilities can be positive or negative. It is well known that stock returns exhibit asymmetric volatility and a negative correlation between stock returns and volatilities can help generate this asymmetric volatility.

B.5. Cross-Market Linkages

This model generates numerous interesting and plausible cross-market dynamics.

$$Cov\left(\frac{dS(t)}{S(t)}, dR(t)\right) = (\sigma_1 - \sigma_1^*)\eta_1\sigma_xX(t) + (\sigma_2 - \sigma_2^*)\eta_2\sigma_yY(t) \tag{30}$$

$$Cov\left(dR(t), dR^*(t)\right) = \eta_1\eta_1^*\sigma_x^2X(t) \tag{31}$$

$$Cov\left(\frac{dP(t)}{P(t)}, \frac{dP^*(t)}{P^*(t)}\right) = \gamma_1\gamma_1^*\sigma_x\sigma_y^*X(t) \tag{32}$$

$$Cov(dV_p(t), dV_p^*(t)) = \gamma_1^2\gamma_1^{*2}\sigma_x^6X(t) \tag{33}$$

$$Cov(dV_R, dV_R^*) = \eta_1^2 \sigma_x^6 \eta_1^{*2} X(t) \tag{34}$$

The covariance between the foreign currency depreciation rate and domestic short rate changes is time-varying and can be positive or negative. The covariance between domestic short rate changes and foreign short rate changes is positive, consistent with empirical evidence. In addition, the covariance between domestic stock returns and foreign stock returns is time-varying and positive if γ_1 and γ_1^* have the same sign. The covariance between domestic stock market volatility and foreign stock market volatility is always positive. The covariance between domestic short rate volatility and foreign short rate volatility is also positive.

B.6. Summary

The proposed affine model offers the following desirable features: (a) Stock prices, short rates, exchange rates, and factor risk premia are jointly determined in this model. This guarantees internal consistency; (b) this model can deliver empirically plausible dynamics of asset returns including time-varying volatilities, time-varying correlations, the level effect, and asymmetric volatility; (c) the proposed model yields analytically tractable and explicit pricing formulas for many equity, interest rate, and currency derivatives as well as cross-asset derivatives (see Bakshi and Madan (2000).)

III. Estimation

In this section, we estimate the proposed affine model using the U.S. and U.K. data via EMM. The estimation proceeds as follows: We first search for an auxiliary model, which describes well the conditional density of the observed data. We approximate this density using the Seminonparametric (SNP) procedure of Gallant and Tauchen (1989). In the second step, we use the EMM to estimate the structural parameters. In this step, the expectations of the

SNP scores computed using data simulated under the structural model are minimized. Finally, the model specification is tested using the χ^2 test of over-identifying restrictions and the t test of individual SNP scores. As a starting point of our estimation, we first look at the univariate series. Our focus, of course, is on the multivariate dynamics.

A. Data and Summary Statistics

Daily data of the S&P 500 index, the FTSE 100 index, the British pound/U.S. dollar spot rates, and the three-month British pound/U.S. dollar forward rates were obtained from Logical Information Machines. Three-month U.S. T-bill rates were obtained from the Federal Reserve Bank of St. Louis web site. The daily Euro-Sterling rates we obtained suffer from a severe stale price problem. We instead use the British pound spot rates, British pound three-month forward rates, and U.S. three-month T-bill rates to back out the U.K. three-month interest rates implied by the covered interest rate parity.[5] The daily data likely suffer from the nonsynchronous trading problem. We nonetheless choose to use daily data because the number of weekly or monthly observations appears insufficient for estimating multivariate SNP models. The data cover the period from October 22, 1985, to February 20, 1997, for a total of 2857 observations.

Interest rates are extremely persistent, so we examine the change of interest rates. In contrast, most existing studies of dynamic term structure models in the literature examine the *level* of interest rates. As pointed out by Duffee and Stanton (2000), the SNP/EMM procedure performs poorly when dealing with persistent series such as interest rates. For stock and currency markets, we examine the stock returns and currency returns. In summary, the five series of interest are daily percentage change of the British pound/U.S. dollar exchange rates

(Δe), daily change of U.S. short rates (Δr), daily change of U.K. short rates (Δr^*), daily S&P 500 index returns (Δp), and daily FTSE 100 index returns (Δp^*).

Panel A of Table 1 presents the summary statistics of the above five series. The average daily stock index returns for both U.S. and U.K. are positive. The average changes of U.S. and U.K. short rates are both negative, indicating that interest rates have declined over the sample period. Not surprisingly, stock markets are more volatile than money markets and currency markets. Consistent with existing evidence, both U.S. and U.K. stock markets exhibit negative skewness. All five series are leptokurtic, albeit to different degrees. Stock index returns are more leptokurtic than interest rate changes and currency returns. The first-order serial autocorrelation coefficients are small and generally not significant. Consistent with prior studies, we find positive autocorrelations of stock index returns. This is likely due to non-synchronous trading. Overall, normality is rejected for each of five series. Currency returns and short rate changes are much closer to normality than stock index returns are.

Cross-correlations between five series are reported in Panel B of Table 1. The correlation between U.S. and U.K. stock markets is positive and economically significant, as is the correlation between U.S. and U.K. money markets. Other correlations are not significant.

B. The SNP Models

In this section, we fit SNP models to five univariate series (Δe, Δr, Δr^*, Δp, Δp^*) and two trivariate series ($\{\Delta e, \Delta r, \Delta r^*\}$, $\{\Delta e, \Delta p, \Delta p^*\}$). The first trivariate series include the British pound returns and U.S. and U.K. short rate changes. The second trivariate series include the British pound returns and U.S. and U.K. stock returns. Naturally, one would also want to examine the pentivariate system that includes all five assets. Unfortunately, estimating the joint distribution of all five assets using the SNP is currently not feasible for realistic specifications. Consequently, we were not able to conduct the SNP/EMM analysis for the five-asset system.

B.1. A Brief Introduction of the SNP

The SNP employs a Hermite polynomial expansion to approximate the conditional density of a multivariate process. The SNP is a nonlinear nonparametric model that directly nests the Gaussian vector-autoregression (VAR) model, the Gaussian Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and the semiparametric GARCH model. The Hermite polynomial expansion allows the SNP to capture deviations from normality as well as conditional heterogeneity.

The eight-digit label "$L_\mu L_g L_r L_p K_z I_z K_x I_x$" for each SNP model denotes the following: L_μ is the order of the VAR specification, L_g is the order of GARCH terms in the conditional standard deviation, L_r is the order of ARCH terms in the conditional standard deviation, L_p is the number of lags included in the conditioning set of the Hermite polynomial, K_z is the order in z of the Hermite polynomial, I_z indicates the suppression of cross terms in z of order equal to or higher than I_z, K_x is the order in x of the coefficient in Hermite polynomial, and finally I_x indicates the suppression of cross terms in x. To determine an appropriate SNP specification within the hierarchy of models, we follow the approach suggested by Gallant and Tauchen (1999) and use the Schwartz Bayes Information Criterion.

B.2. Modifications to the SNP

We make two slight modifications to the SNP. First, we constrain the sum of the GARCH and ARCH parameters to be strictly less than one. This is a standard stationarity condition for GARCH models. However, the GARCH specification in the SNP is different from standard

GARCH models in that the SNP models the standard deviation rather than the variance. Although we feel the above constraint is likely too restrictive, we nonetheless impose this constraint to be conservative.

In the SNP modeling of multivariate series, Gallant and Tauchen (1999) recommend that one use diagonal GARCH models (i.e., nondiagonal terms are constants) and allow the Hermite expansion to capture the cross-series dynamics. In this paper, we relax the orthogonality constraint by allowing the covariances to have a GARCH term but no ARCH terms. This dramatically reduces the number of parameters as compared to the nondiagonal GARCH. At the same time, it provides a parsimonious AR(1) model for the covariances.

B.3. SNP Results

We start with the estimation of the conditional densities of five univariate series. Space constraints preclude us from reporting the objective function values of all the models considered. Table 2 reports estimates of selected SNP parameters for the final preferred models. Roughly speaking, the ψs are VAR parameters that are associated with the first moment. The τs are GARCH parameters and are associated with the second moment. Finally, the As are parameters in the Hermite polynomial expansion and are generally associated with the higher moments.

British Pound (Δe): 01114010. This model is similar to a GARCH(1,1) with a time-varying, thick-tailed error density, as reflected by the dependence of the Hermite coefficients on lagged returns (K_x=1.) This model has 13 parameters. There is little predictability in the mean, as it is best captured by a constant. The volatility of the British pound is very persistent, with the sum of the ARCH and GARCH parameters being 0.9628.

U.S. short rate (Δr): 21114000. This model is an AR(2) with a GARCH(1,1) and a timehomogenous, thick-tailed error density. This model contains 10 parameters. The changes of U.S. short rates are positively autocorrelated. This positive autocorrelation is consistent with the mean reversion of interest rates and the fact that the Federal Reserve Bank tends to adjust interest rates in successive small steps. The volatility of the U.S. short rate is also very persistent, with the sum of the ARCH and GARCH parameters being 0.9743.

U.K. short rate (Δr^*): 21114000. This model is same as that of U.S. short rates. In contrast to the U.S. short rate changes, the U.K. short rate changes are negatively autocorrelated. The volatility of the U.K. short rate is much more persistent than that U.S. short rates. Unconstrained, the ARCH and GARCH parameters would add up to 1.037.

S&P 500 index return (Δp): 11118010. This model is an AR(1) with a GARCH(1,1) and time-varying shape characteristics. This model has 22 parameters. The sum of the ARCH and GARCH parameters totals 0.999, equaling the constraint we imposed on this sum.

FTSE 100 Index return (Δp^*): 11116010. This model is an AR(1) with a GARCH(1,1) conditional variance and time-varying shape characteristics. This model has 18 parameters. The FTSE 100 index return volatility is less persistent than the S&P 500 index return.

We also fit SNP models to two trivariate series that are described at the beginning of Section II.B. In each trivariate series, the final SNP model selected is 11114300. Table 3 presents the SNP estimates of these two models. Because the primary focus of our paper is on the *joint* dynamics of international asset returns, it is useful to distinguish between those SNP parameters that govern the cross-asset dynamics and those SNP parameters that are associated with only one asset. Specifically, in our preferred model 11114300, all the interaction terms in the Hermite polynomial expansion are suppressed. Thus none of the As are directly related to cross-asset dynamics. On the other hand, ψ_5, ψ_6, ψ_7, ψ_9, ψ_{10}, and ψ_{11} are associated with cross-asset relations in the first moment. In addition, τ_2, τ_4, τ_5, τ_{26}, τ_{28}, and τ_{29} are associated with cross-asset relations in the second moment. In Table 3, we see that many of these parameters are statistically significant, indicating significant cross-asset dynamics.

One way to ensure that the selected SNP model captures the basic features of the data is to simulate data from the selected SNP model and then compare the summary statistics of these simulated data with that of actual data. Unreported results from this exercise indicate that all selected SNP models match the first, second, and third moments quite well. The multivariate SNP models also match the cross-asset correlations remarkably well. However, none of these SNP models generate enough kurtosis. We note that the amount of kurtosis in the actual data is greatly influenced by a couple of "outliers," especially the crash of 1987. Removing these outliers, the SNP models fit the fourth moments fine.

C. EMM Estimation

C.1. A Brief Introduction of the EMM

Gallant and Tauchen (1996) present a systematic approach to generating moment conditions for the generalized method of moments estimator. Their idea is to use the expectation under the structural model of the score from an auxiliary model as the vector of moment conditions. The score is the derivative of the log density of the auxiliary model with respect to the parameters of the auxiliary model.

When a model fails the specification test, one would like some indications as to what is wrong. Inspection of the quasi-t-ratios of SNP scores can suggest reasons for model failure. Different elements of the score vector correspond to different features of the fit. Large quasi-t-ratios reveal the features of the data that the maintained model can not approximate.

The implementation of EMM requires simulating data from the structural model. We use the Euler scheme. We divide each day into four equal intervals and obtain 50,000 daily observations (roughly 200 years of data) by aggregating every four observations from 200,000 simulated ones. The estimation is done on an Hewlett-Packard workstation. The FORTRAN programs used to implement the SNP and EMM were downloaded from Ron Gallant's web site.

C.2. Univariate Results

Univariate estimation results are reported in Table 4 for Δr, Δr^*, Δp and Δp^*. Estimating the proposed model for Δe via EMM is not feasible because there are more parameters (16) in the structural model than that in the corresponding SNP (13). Recall also that there are three factors in the model, two of which are local factors each affecting only one country. So basically, we are estimating a two-factor model for each of the four univariate series.

The first column of Table 4 presents the EMM estimation results for Δr. The second, third, and fourth columns present the estimation results for Δr^*, Δp, and Δp^*, respectively. All parameter estimates are annualized and based on data in percent. The long-run means of X, Y, and Z are not separately identified, so we arbitrarily set all of them to 1. This effectively normalizes all the structural parameters. We also set starting values of X, Y, and Z to 1 in the simulation. In addition, all structural parameters except γs and γ^*s are constrained to be positive to ensure positive interest rates, positive volatilities, and positive speed of mean reversion.

The last three rows of Table 4 report the χ^2 test of over-identifying restrictions, the degree of freedom for the χ^2 test, and the z statistic, respectively. The z statistic is transformed from the χ^2 statistic and is distributed as a standard normal asymptotically. In discussions below, we focus on z statistics because they are easier to interpret. The z statistic for Δr is 25.61, indicating a strong rejection of the model. The z statistics for Δr^*, Δp, and Δp^* are 11.42, 17.68, and 5.22, respectively. In each case, the model is rejected at the 1% level. Relatively speaking, the proposed model fits the FTSE 100 index and the U.K. short rate better than the S&P 500 index and U.S. short rates. This is as expected. Recall from Table 1 that the unconditional distribution of the FTSE 100 index and the U.K. short rate are less skewed and less leptokurtic than their U.S. counterparts.

Examining individual t-ratios associated with each moment conditions provides us additional insight into the performance of model. Roughly speaking, the t-ratios for As indicate the fit for higher order moments of the conditional density and departures from the normal distribution. The t-ratios for ψs represent the fit for first moments of the conditional density. Finally, the t-ratios for τs indicate the fit for second moments.

Table 5 presents these t-ratios. We find that the proposed model generally has little problem fitting the volatility dynamics (indicated by low t-ratios on τs) but has difficulty fitting the higher order moments (indicated by high t-ratios on As.) The t ratios on A_3, A_5, and A_9 are particularly high. Because A_3, A_5, and A_9 represent the quadratic and quartic terms in the Hermite polynomial, the above results indicate that the model has particular trouble matching the kurtosis. In addition, the model has trouble fitting the first moments of the short rate changes. This is likely due to the nonlinearity in the drift of interest rate processes (Ait Sahalia (1996)).

C.3. Trivariate Results

In this section we conduct tests of goodness-of-fit for two trivariate series: $\{\Delta e, \Delta r, \Delta r^*\}$ and $\{\Delta e, \Delta p, \Delta p^*\}$. Table 6 presents the parameter estimates and specification test results. The last rows of the table present the χ^2 statistics and the z statistics for model fit. The z statistic for $\{\Delta e, \Delta r, \Delta r^*\}$ is 3.74, suggesting a rejection of overidentifying restrictions implied by the model at the 1% level. The z statistic for $\{\Delta e, \Delta p, \Delta p^*\}$ is 48.72, meaning that the model is strongly rejected. Clearly, the model does a much better job fitting $\{\Delta e, \Delta r, \Delta r^*\}$ than $\{\Delta e, \Delta p, \Delta p^*\}$. This might be due to the greater departure from normality of the distribution of stock index returns than that of short rate changes. Another possibility is that both the exchange rates and interest rates are influenced by common factors such as macroeconomic and monetary variables, whereas the stock returns seem to be driven by a separate set of factors.

Diagnostic t-ratios on SNP scores are reported in Table 7. Inspection of these t-ratios reveal that the model does an adequate job fitting the second moments of $\{\Delta e, \Delta r, \Delta r^*\}$, with 3 out of 21 t-ratios on τs being slightly greater than 2 in absolute value. The model has difficulty fitting the higher order moments, with 3 out of 12 t-ratios on As greater than 2 in absolute value. Consistent with results from univariate analysis, the model continues to experience difficulty with the first moments of short rate changes, as reflected by high t-ratios on ψ_2 and ψ_8. As for $\{\Delta e, \Delta p, \Delta p^*\}$, t-ratios are generally higher and there are more significant t-ratios when compared to $\{\Delta e, \Delta r, \Delta r^*\}$. The model has immense difficulty with higher order moments, with 6 out 12 t-ratios highly significant. Additionally, many of the t-ratios on τs are significant, indicating the model's inability to fit the second moments.

A closer inspection of t-ratios of both series reveals that the model does a good job of fitting cross-asset dynamics, as indicated by relatively low t-ratios for ψ_5, ψ_6, ψ_7, ψ_9, ψ_{10} , ψ_{11}, τ_2, τ_4, τ_5, τ_{26}, τ_{28}, and τ_{29}. This result suggests that the model, despite experiencing difficulty with certain features of the data, has little trouble capturing the cross-asset dynamics.

One possible reason that the proposed model is rejected is that three factors (one common factor and two local factors) are not enough to describe the joint distribution of exchange rates, short rates, and stock returns. We extend the model to allow for two extra local factors. We repeat the goodness-of-fit tests. Results are reported in Table 8. Notice that even though we have a total of five factors, each of the short rates and stock indexes are driven by only three factors. Not surprisingly, there is dramatic improvement in model fit with both series. As in the case of three factors, the model does a much better job fitting $\{\Delta e, \Delta r, \Delta r^*\}$ than $\{\Delta e, \Delta p, \Delta p^*\}$. The z statistic is now just 2.29 for $\{\Delta e, \Delta r, \Delta r^*\}$, indicating that we cannot reject the model at the 1% level. By looking at t-ratios on SNP scores, we find that the model does a good job overall. The one area in which the model needs improvement is the first moment of short rate changes. As for $\{\Delta e, \Delta p, \Delta p^*\}$, the z statistic drops from 48.72 to 19.79. Examining the t-ratios on SNP scores reveals that the model now does a much better job fitting second moments but continues to have trouble fitting the higher order moments of stock index returns.

IV. Conclusion

Existing empirical studies of affine models have focused on the term structure of interest rates, stock indexes, or currencies, usually in isolation. In this paper, we investigate whether affine models do an adequate job of fitting the joint dynamics of international stock, bond, and currency prices. It is common practice to model multivariate series by independently developing models for each univariate time series and assembling them in some fashion. A presumably better alternative is to use a single, integrated model. To this end, we develop a two-country affine model that can simultaneously price stocks, bonds, and currencies. By doing so, internal consistency is guaranteed. The proposed affine model, despite its simple form, is capable of reproducing many empirical stylized facts.

We estimate the proposed affine model using the SNP/EMM framework. Estimation results from univariate series show that the model does a good job of fitting the first and second moments, but it has difficulty fitting the higher order moments. Additionally, the positive autocorrelation in U.S. short rate changes and negative autocorrelation in U.K. short rate changes are difficult to fit. This is perhaps due to the nonlinearity in the interest rate process. Estimation results from the trivariate analysis reveal that affine models do a good job fitting cross-asset dynamics, but again have difficulties with higher order moments and some first and second moments. We also find that the proposed model does a much better job fitting the joint dynamics of currency returns and short rate changes than that of currency returns and stock returns. We conjecture that this is explained by two factors. First, stock index returns are more leptokurtic than short rate changes. Second, exchange rates and interest rates share many common factors such as macroeconomic and monetary variables, whereas stock returns seem to be driven by a separate set of factors.

The empirical performance of the five-factor model is substantially better than that of the three-factor model, especially in the second moments, but simply adding more factors is not the solution to all the difficulties that affine models face. Our results suggest areas in which affine models need to be improved. For instance, the fact that the proposed affine models have trouble generating sufficient short-run leptokurtosis suggests that allowing for random jumps along the lines of Bates (1996, 2000) will help. At present, estimating multivariate jump diffusion models is not yet feasible. Developing reliable, easy-to-implement, and efficient estimators for multivariate jump diffusion processes is hence called for in future research.

Footnotes

1. See Bates (1996), Ahn (1997), and Bakshi and Chen (1997a) for currencies and Schwartz (1997) for commodities.

2. Competing models, including regime-switching models, constant elasticity of variance models, and quadratic models, are generally not as tractable as affine models. Ahn, Dittmar, and Gallant (2001) obtain closed-form bond prices for quadratic models, but it remains to be seen whether these models are as tractable as affine models in pricing bond derivatives. It is also not clear to what extent the quadratic term structure models are useful for modeling stocks and currencies.

3. See Bakshi and Chen (1997b) for details.

4. This is analogous to the aggregation of ARMA processes. For instance, the sum of two independent AR(1) process follows a ARMA(2,1), which has a more slowly decaying autocorrelation function than a AR(1) does.

5. $r_t^* = (1 + r_t)\left(\frac{F_{t,90}}{S_t}\right)^{-4} - 1$, where r_t^* is the three-month U.K. risk-free rate, r_t is the U.S. three-month T-bill rate, S_t is the spot rate of British pound, and $F_{t,90}$ is the three-month British pound forward rate.

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TABLE 1
Summary Statistics

Δe is the percentage change of British pound/U.S. dollar exchange rates. Δr is the change of U.S. three-month T-bill rates. Δr^* is the change of U.K. three-month interest rates. Δp is the percentage S&P 500 index return. Δp^* is the percentage FTSE index return. All series are daily and in percent. ρ_1 is the first-order serial autocorrelation coefficient. The normality test employed is the Jarque-Bera test. The data cover the period from October 22, 1985, to February 20, 1997, for a total of 2857 observations.

Panel A: Summary Statistics

Series	Δe	Δr	Δr^*	Δp	Δp^*
mean	0.0066	-0.0008	-0.0017	0.0558	0.0455
median	0.0000	0.0000	-0.0036	0.0668	0.0521
maximum	3.4333	0.4400	0.7329	9.0990	7.8930
minimum	-3.8193	-0.6200	-1.0801	-20.4566	-12.2156
standard deviation	0.6921	0.0606	0.1259	0.9684	0.9216
skewness	-0.2095	-0.2600	-0.0419	-3.5689	-1.3609
kurtosis	5.8057	14.3690	9.5814	80.2396	24.7094
ρ_1	0.0270	0.1450	-0.0030	0.0370	0.0480
p value (H_0: normal)	0.0000	0.0000	0.0000	0.0000	0.0000

Panel B: Correlation Matrix

Series	Δe	Δr	Δr^*	Δp	Δp^*
Δe	1.0000	-0.0484	-0.2000	-0.0765	-0.0761
Δr		1.0000	0.4136	-0.0237	0.0818
Δr^*			1.0000	0.0819	-0.0605
Δp				1.0000	0.4002
Δp^*					1.0000

TABLE 2
SNP Estimates: Univariate Series

Estimates of final preferred SNP models are reported for five univariate series. Δe is the percentage change of British pound/U.S. dollar exchange rates. Δr is the change of U.S. three-month T-bill rates. Δr^* is the change of U.K. three-month interest rates. Δp is the S&P 500 index return. Δp^* is the FTSE index return. ψ_1, ψ_2, and ψ_3 are VAR parameters. The τs are conditional volatility parameters. τ_1 is the constant. τ_2 is the ARCH parameter. τ_3 is the GARCH parameter. The As are parameters in Hermite polynomial expansion. Standard errors are given in parentheses. Data cover the period from October 22, 1985, to February 20, 1997, for a total of 2857 observations. Empty cells imply inapplicability.

	Δe	Δr	Δr^*	Δp	Δp^*
SNP Model	01114010	21114000	21114000	11118010	11116010
A_5	-0.1503	0.0074	0.0076	-0.2828	-0.3842
	(0.0136)	(0.0039)	(0.0043)	(0.0266)	(0.0218)
A_9	0.0231			0.0819	0.0588
	(0.0018)			(0.0117)	(0.0070)
A_{13}				-0.0090	-0.0027
				(0.0017)	(0.0006)
A_{17}				(0.0003)	
				(0.0001)	
ψ_1	0.0005	-0.0443	0.0387	0.0776	0.0787
	(0.0357)	(0.0129)	(0.0215)	(0.0351)	(0.0413)
ψ_2		-0.0678	-0.0728	0.2223	0.0502
		(0.0183)	(0.0163)	(0.0312)	(0.0422)
ψ_3		0.0858	-0.0789		
		(0.0186)	(0.0188)		
τ_1	0.0650	0.0687	0.0476	0.0677	0.1330
	(0.0052)	(0.0041)	(0.0047)	(0.0097)	(0.0171)
τ_2	0.1147	0.1569	0.1624	0.23719	0.1703
	(0.0121)	(0.0103)	(0.0105)	(0.0108)	(0.0195)
τ_3	0.8481	0.8174	0.8366	0.8036	0.8201
	(0.0100)	(0.0086)	(0.0090)	(0.0162)	(0.0178)
$\tau_2 + \tau_3$	0.9628	0.9743	0.9990	0.9990	0.9904

TABLE 3
SNP Estimates: Trivariate Series

Estimates of final preferred SNP models are reported for two trivariate series. Δe is the percentage change of British pound/U.S. dollar exchange rates. Δr is the change of U.S. three-month T-bill rates. Δr^* is the change of U.K. three-month interest rates. Δp is the S&P 500 index return in percent. Δp^* is the FTSE index return in percent. The ψs are VAR parameters. The τs are conditional volatility parameters. τ_1 is the constant. τ_2 is the ARCH parameter. τ_3 is the GARCH parameter. The As are parameters in Hermite polynomial expansion. Standard errors are given in parentheses. Data cover the period from October 22, 1985, to February 20, 1997, for a total of 2857 observations.

Parameter	$\{\Delta e, \Delta r, \Delta r^*\}$	$\{\Delta e, \Delta p, \Delta p^*\}$	Parameter	$\{\Delta e, \Delta r, \Delta r^*\}$	$\{\Delta e, \Delta p, \Delta p^*\}$
A_2	-0.029 (0.017)	0.011 (0.020)	ψ_{12}	-0.008 (0.011)	-0.006 (0.020)
A_3	-0.012 (0.012)	-0.287 (0.018)	τ_1	0.076 (0.005)	0.066 (0.005)
A_4	0.035 (0.010)	0.005 (0.017)	τ_2	-0.014 (0.023)	-0.022 (0.027)
A_5	-0.244 (0.011)	-0.245 (0.019)	τ_3	0.191 (0.012)	0.283 (0.012)
A_6	-0.131 (0.010)	-0.034 (0.012)	τ_4	-0.217 (0.074)	-0.056 (0.098)
A_7	-0.069 (0.007)	-0.122 (0.010)	τ_5	0.100 (0.022)	-0.013 (0.007)
A_8	0.033 (0.003)	0.004 (0.004)	τ_6	0.414 (0.027)	0.203 (0.016)
A_9	-0.019 (0.002)	0.033 (0.003)	τ_7	0.096 (0.009)	0.117 (0.012)
A_{10}	-0.002 (0.003)	-0.001 (0.003)	τ_8	-0.029 (0.024)	0.021 (0.025)
A_{11}	0.016 (0.002)	0.014 (0.002)	τ_{10}	0.093 (0.025)	0.006 (0.031)
A_{12}	0.003 (0.001)	0.000 (0.001)	τ_{14}	0.093 (0.025)	-0.034 (0.021)
A_{13}	0.017 (0.001)	0.019 (0.001)	τ_{15}	0.215 (0.013)	0.165 (0.009)
ψ_1	-0.031 (0.019)	-0.011 (0.030)	τ_{17}	0.029 (0.011)	0.117 (0.012)
ψ_2	0.057 (0.018)	0.384 (0.027)	τ_{22}	-0.041 (0.029)	-0.028 (0.037)
ψ_3	-0.023 (0.025)	-0.023 (0.026)	τ_{23}	0.123 (0.016)	0.008 (0.013)
ψ_4	-0.018 (0.019)	-0.000 (0.020)	τ_{24}	0.180 (0.012)	0.210 (0.013)
ψ_5	-0.019 (0.011)	0.017 (0.015)	τ_{25}	0.826 (0.011)	0.844 (0.011)
ψ_6	0.027 (0.010)	0.021 (0.014)	τ_{26}	0.279 (0.207)	0.487 (0.329)
ψ_7	0.002 (0.018)	0.006 (0.020)	τ_{27}	0.682 (0.015)	0.579 (0.014)
ψ_8	0.063 (0.015)	0.008 (0.020)	τ_{28}	0.205 (0.272)	0.467 (0.840)
ψ_9	-0.067 (0.009)	0.232 (0.016)	τ_{29}	0.618 (0.052)	0.839 (0.024)
ψ_{10}	0.007 (0.018)	0.007 (0.018)	τ_{30}	0.556 (0.026)	0.736 (0.016)
ψ_{11}	-0.022 (0.012)	0.008 (0.016)			

TABLE 4
EMM Estimates: Univariate Series

Parameter estimates and goodness-of-fit test statistics for the proposed model described in Section I.B are presented for four univariate series. Δr is the change of U.S. three-month T-bill rates. Δr^* is the change of U.K. three-month interest rates. Δp is the S&P 500 index return. Δp^* is the FTSE index return. Standard errors are given in parentheses. Empty cells imply inapplicability. The z statistic is based on the χ^2 statistic adjusting for the degree of freedom and is distributed as a standard normal.

	Δr	Δr^*	Δp	Δp^*
κ_x	3.0313(1.2220)	0.5499(0.3630)	0.5944(0.1425)	0.1699(0.0538)
σ_x	1.0847(0.2606)	0.1426(0.0179)	0.7704(0.2853)	0.2162(0.1077)
κ_y	0.2243(0.0994)		2.0315(0.4061)	
σ_y	0.0114(0.1114)		0.1797(0.0815)	
κ_z		2.6710(0.8051)		0.3040(0.2710)
σ_z		0.1637(0.1690)		0.7307(0.5085)
η_1	0.3806(0.2760)			
η_2	5.7422(2.6002)			
η_1^*		2.1362(0.2470)		
η_2^*		1.6488(0.8337)		
γ_1			0.1410(0.0143)	
γ_2			0.0303(0.0041)	
γ_1^*				0.0967(0.5383)
γ_2^*				0.1781(0.0261)
χ^2	76.434	36.304	115.993	37.591
df	4	4	16	12
z	25.61	11.42	17.68	5.22

TABLE 5
Diagnostic t-ratios of SNP Scores: Univariate Models

Diagnostic t-ratios of SNP scores are presented for four univariate series. The As are Hermite polynomial parameters. The τs are conditional volatility parameters. The ψs are VAR parameters. Δr is the change of U.S. three-month T-bill rates. Δr^* is the change of U.K. three-month interest rates. Δp is the S&P 500 index return in percent. Δp^* is the FTSE index return in percent. Empty cells imply inapplicability.

	Δr	Δr^*	Δp	Δp^*
A_2	-0.582	-0.713	-1.509	0.813
A_3	0.630	-0.141	4.080	0.402
A_4	1.125	0.683	2.616	1.125
A_5	3.251	1.572	3.926	-1.179
A_6			-2.678	0.531
A_7			1.268	0.185
A_8			1.948	-0.102
A_9			4.512	-2.423
A_{10}			-1.995	0.468
A_{11}			-0.657	0.440
A_{12}			1.352	-0.616
A_{13}			3.840	-1.949
A_{14}			-1.472	0.190
A_{15}			-1.577	
A_{16}			1.088	
A_{17}			3.164	
A_{18}			-1.152	
ψ_1	-1.398	-0.468	1.186	1.731
ψ_2	-1.474	-2.531	1.982	1.236
ψ_3	3.493	-2.057		
τ_1	1.282	0.525	1.476	-0.834
τ_2	-0.729	0.086	1.214	-0.478
τ_3	0.633	0.778	1.196	-0.845

TABLE 6
EMM Estimates: Trivariate Series

Parameter estimates and goodness-of-fit test statistics for the proposed model described in Section I.B are presented for two trivariate series. Δe is the percentage change of British pound/U.S. dollar exchange rates. Δr is the change of U.S. three-month T-bill rates. Δr^* is the change of U.K. three-month interest rates. Δp is the S&P 500 index return in percent. Δp^* is the FTSE index return in percent. Standard errors are given in parentheses. Empty cells imply inapplicability. The z statistic is based on the χ^2 statistic adjusting for the degree of freedom and is distributed as a standard normal.

	$\{\Delta e, \Delta r, \Delta r^*\}$	$\{\Delta e, \Delta p, \Delta p^*\}$
κ_x	0.1352 (0.0277)	0.4116 (0.6357)
σ_x	0.2456 (0.0254)	0.4562 (0.3523)
κ_y	0.1197 (0.0595)	0.4498 (0.0219)
σ_y	0.0137 (0.1171)	0.3422 (0.0068)
κ_z	1.0034 (0.8373)	0.3119 (0.0051)
σ_z	0.1687 (0.0721)	0.4172 (0.0020)
μ_m	0.8936 (0.6636)	1.1496 (1.5649)
η_1	6.6040 (6.8014)	0.6979 (0.4081)
η_2	0.4873 (0.2272)	0.5768 (0.2218)
μ_m^*	0.8721 (0.2144)	1.1436 (1.5648)
η_1^*	7.3482 (1.3465)	0.7440 (0.2835)
η_2^*	0.0789 (0.6065)	0.0100 (0.8088)
σ_1	0.9928 (1.4641)	0.6352 (0.7935)
σ_1^*	0.9949 (1.4639)	0.9380 (0.6857)
σ_2	0.8613 (0.2877)	0.2508 (0.9299)
σ_2^*	0.9459 (0.2878)	0.0102 (2.9909)
σ_3	1.0764 (0.3556)	0.8147 (5.2615)
σ_3^*	1.0949 (0.3556)	0.9059 (5.2603)
γ_1		0.4334 (6.9431)
γ_2		0.4184 (1.5484)
γ_1^*		0.3890 (6.0087)
γ_2^*		0.6010 (1.7221)
χ^2	54.462	353.453
df	27	23
z	3.74	48.72

TABLE 7
Diagnostic t-ratios of SNP Scores on Trivariate Series: The Three-factor Model

Diagnostic t-ratios of the SNP scores are presented for two trivariate series. The As are Hermite polynomial parameters. The τs are conditional volatility parameters. The ψs are VAR parameters. Δe is the percentage change of British pound/U.S. dollar exchange rates. Δr is the change of U.S. three-month T-bill rates. Δr^* is the change of U.K. three-month interest rates. Δp is the S&P 500 index return in percent. Δp^* is the FTSE index return in percent. Empty cells imply inapplicability.

Parameter	$\{\Delta e, \Delta r, \Delta r^*\}$	$\{\Delta e, \Delta p, \Delta p^*\}$	Parameter	$\{\Delta e, \Delta r, \Delta r^*\}$	$\{\Delta e, \Delta p, \Delta p^*\}$
A_2	0.287	-1.897	ψ_{12}	-0.365	2.845
A_3	1.502	1.945	τ_1	-0.708	-2.821
A_4	1.878	-2.731	τ_2	0.852	-1.783
A_5	2.842	-7.093	τ_3	1.571	-1.024
A_6	1.193	0.052	τ_4	2.496	-0.062
A_7	-2.841	-5.689	τ_5	1.312	2.262
A_8	0.908	-1.667	τ_6	1.370	-1.414
A_9	-1.475	0.450	τ_7	-0.950	-3.858
A_{10}	-0.173	-10.520	τ_8	0.146	-8.182
A_{11}	0.362	-7.193	τ_{10}	1.402	4.732
A_{12}	0.101	1.021	τ_{14}	0.717	-0.920
A_{13}	4.061	2.010	τ_{15}	1.809	-0.611
ψ_1	1.030	1.590	τ_{17}	1.666	-1.334
ψ_2	5.191	8.742	τ_{22}	0.993	8.710
ψ_3	-1.399	1.249	τ_{23}	1.326	-1.820
ψ_4	-1.263	-0.538	τ_{24}	1.175	-4.181
ψ_5	-0.767	-1.838	τ_{25}	-1.251	2.016
ψ_6	1.351	-2.200	τ_{26}	0.244	0.985
ψ_7	-0.216	1.516	τ_{27}	2.539	1.656
ψ_8	3.105	-6.206	τ_{28}	-1.463	-0.989
ψ_9	-0.785	0.307	τ_{29}	2.049	2.303
ψ_{10}	0.195	0.833	τ_{30}	1.905	-4.636
ψ_{11}	0.242	-1.839			

TABLE 8
Diagnostic t-ratios of SNP Scores on Trivariate Series: The Five-factor Model

Diagnostic t-ratios of the SNP scores are presented for two univariate series. The As are Hermite polynomial parameters. The τs are conditional volatility parameters. The ψs are VAR parameters. Δe is the percentage change of British pound/U.S. dollar exchange rates. Δr is the change of U.S. three-month T-bill rates. Δr^* is the change of U.K. three-month interest rates. Δp is the S&P 500 index return in percent. Δp^* is the FTSE index return in percent. Empty cells imply inapplicability. The z statistic is based on the χ^2 statistic adjusting for the degree of freedom and is distributed as a standard normal.

Parameter	$\{\Delta e, \Delta r, \Delta r^*\}$	$\{\Delta e, \Delta p, \Delta p^*\}$	Parameter	$\{\Delta e, \Delta r, \Delta r^*\}$	$\{\Delta e, \Delta p, \Delta p^*\}$
A_2	-0.155	0.799	ψ_{12}	-0.082	-0.039
A_3	0.751	5.462	τ_1	-0.091	-1.379
A_4	0.604	1.649	τ_2	0.092	0.540
A_5	1.463	-0.519	τ_3	0.956	-5.353
A_6	0.944	-2.220	τ_4	0.005	0.201
A_7	-0.047	-4.542	τ_5	-0.736	-0.270
A_8	0.696	0.102	τ_6	0.364	-0.329
A_9	-0.267	1.718	τ_7	-0.100	-0.397
A_{10}	-0.126	1.323	τ_8	0.004	-0.208
A_{11}	0.269	-0.103	τ_{10}	0.018	0.020
A_{12}	0.108	-0.735	τ_{14}	0.012	-0.176
A_{13}	-0.405	-4.138	τ_{15}	0.192	-0.303
ψ_1	-0.140	0.167	τ_{17}	-0.500	0.301
ψ_2	2.024	4.165	τ_{22}	0.002	-0.010
ψ_3	-1.245	0.878	τ_{23}	0.723	-0.361
ψ_4	-0.141	-0.035	τ_{24}	-0.808	0.684
ψ_5	-0.167	-0.296	τ_{25}	0.365	1.106
ψ_6	-0.554	-0.342	τ_{26}	-0.100	0.417
ψ_7	-0.026	0.054	τ_{27}	1.708	1.346
ψ_8	0.097	0.202	τ_{28}	-0.238	0.402
ψ_9	-0.097	0.616	τ_{29}	1.140	-0.178
ψ_{10}	-0.024	0.002	τ_{30}	0.483	1.205
ψ_{11}	0.078	-0.203	χ^2	30.362	103.842
df	17	11	z	2.29	19.79